Cherenkov Radiation

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1 Introduction

The Russian physicists Cherenkov, Frank, and Tamm, received the 1964 Nobel prize in physics for their work on Cherenkov radiation.

2 Wave Motion

An example of a wave is the periodic sin function, as a function of time,

$$\sin(\omega t).$$

A traveling wave is a function of both space and time, for example

$$\sin(kx - \omega t),$$

or

$$\psi(x,t) = a\exp(i(kx - \omega t)).$$

Because every periodic function may be expanded in a Fourier series, we may consider only the trigonometric functions of the form

$$\psi(x,t) = a\exp(i(kx - \omega t)).$$

Here k is the wave number and ω is the angular frequency. The wave length is the period of the function with respect to time. For a fixed time t, the wave length λ is the spacial distance between corresponding phase points, that is phase points where

$$\psi(x + \lambda, t_0) = \psi(, t_0)$$

So that, for example,

$$\cos(k(x + \lambda) - \omega t_0) = \cos(kx - \omega t_0)$$

or

$$k(x + \lambda) - \omega t_0 - (kx - \omega t_0) = 2\pi$$

or

$$k\lambda = 2\pi.$$

So

$$\lambda = \frac{2\pi}{k}.$$

The time period T is the time between corresponding phase points when x is held fixed. So in a manner similar to the wavelength calculation, we find the period to be

$$T = \frac{2\pi}{\omega}.$$

The frequency is the reciprocal of the period

$$f = \frac{1}{T}.$$

The phase velocity is the speed that a point on the traveling wave moves. It moves one wavelength λ in period T. Thus the so called phase velocity, being the velocity of a point at a givin phase in the cycle, is

$$v = \frac{\lambda}{T} = \frac{2\pi/k}{2\pi/\omega} = \frac{\omega}{k} = \lambda f.$$

3 The Phase and Group Velocity of Waves

The concept of group velocity was first treated by Lord Rayleigh. According to a reference, a discussion may be found in Rayleigh's **The Theory of Sound**, (1945 Dover reprint of the 1896 Edition). The book has no index, so good luck finding the discussion. In a dispersive medium, waves of different wave lengths travel at different velocities and this results in for example the refraction of light where white light is broken up into a spectrum as it passes through a prism. So suppose a wave packet consists of a sum of waves of different phase velocities.

$$\psi(x,t) = \int_{-\infty}^{\infty} a(k) e^{i(kx-\omega t)} dk,$$

where $\omega(k)$ is a function of the wave number k.

Suppose that the amplitude $a(k)$ is negligable outside of an interval Δk, then the integral becomes approximately

$$\psi(x,t) = \int_{\Delta k} a(k) e^{i(kx-\omega(k)t)} dk.$$

If we approximate $\omega(k)$ by a first order Taylor expansion about k_0 we write approximately

$$\psi(x,t) = e^{k_0 x - \omega_0 t} \int_{\Delta k} a(k) e^{i(x-(d\omega/dk)t)(k-k_0)} dk.$$

where a wave of velocity

$$v = \frac{\omega_0}{k_0},$$

is modulated by an amplitude that moves at the group velocity

$$v_g = \frac{d\omega}{dk}$$

So wave packets move at the group velocity, which is an important concept in quantum mechanics. Refer to Powell and Crasemann.

See the Wikipedia article **Group Velocity** for an animation.

4 Shock Waves

Shock waves are generated when a disturbing object or particle moves through a medium at a velocity greater than the phase velocity of the waves in that medium, which are generated by the disturbing object. Thus an airplane traveling faster than the speed of sound generates shock waves in the atmosphere. This phenomenon produces the familiar sonic boom.

When charged particles such as electrons move through a medium faster than the velocity of light in that medium, they generate electromagnetic waves. These photons or waves produce a shock wave called Cherenkov radiation (or cerenkov). See the figure.

5 The Theory and Spectrum of Cherenkov Radiation

The theory of Cherenkov radiation was given by Cherenkov, and by Frank and Tam in Russian publications in the 1930's, (See the bibliography). The theory of Cherenkov radiation is sketched by Jelly in the Encyclopedia article given in the bibliography. The passage of the particle generates a polarization in the medium, which switches as the particle passes by, and creates a dipole moment that creates dipole radiation. Because of the Cherenkov shock wave angle constraint involving the index of refraction n of the medium, and the ratio β of the two velocities, the shockwave exists only for wavelengths smaller than a certain value λ_2. And because of a relativity constraint there is a lower limit λ_1 also. Thus the radiation is confined to an interval $[\lambda_1, \lambda_2]$, but there is a continuous spectrum in this interval. Using this information it is possible to detect the velocity of the particle, and the mass of the particle, given its energy measured by some other means.

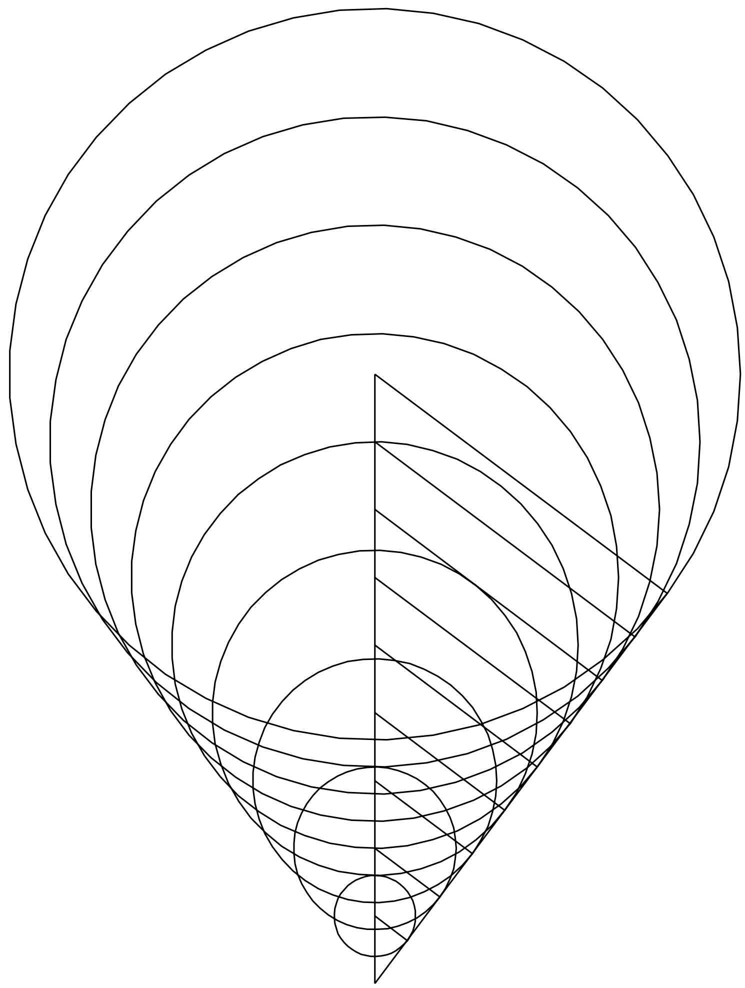

Figure 1: A particle moves down the vertical line. It emits a light wave at the top of the line at a velocity a fraction of the velocity of the particle, and the wave front becomes the large circle by the time the particle reaches the bottom of the vertical path. Likewise light is emitted all along the vertical path each becoming a circle by the time the particle reaches the bottom of the path. The circular waves overlap in a shock front moving out at an angle to the particle path.

Green in (See bibliography) has a simple outline of the theory in an appendix.

Jackson (See bibliography) treats Cherenkov radiation in chapter 13 of the 1974 edition, pp638-641.

6 Nuclear Reactors

The blue glow that one sees in looking down into the water covering a nuclear reactor core, is Cherenkov radiation.

7 Cosmic Rays

Here is a reference to an authorative cosmic ray review created by the Particle Data Group.

`http://pdg.lbl.gov/2005/reviews/cosmicrayrpp.pdf`

The following is from a Wikipedia article on cosmic rays. I find it suspicious.

"Cosmic rays consist of primary cosmic rays and secondary cosmic rays. Secondary rays are produced in the interaction with the primary rays of stellar or planatary atmospheres. Primary cosmic rays are energetic particles originating from outer space that impinge on Earth's atmosphere. Almost 90 percent of all the incoming cosmic ray particles are protons, about 9 percent are helium nuclei (alpha particles) and about 1 percent are electrons (beta minus particles).

The variety of particle energies reflects the wide variety of sources. The origins of these particles range from energetic processes on the Sun all the way to as yet unknown events in the farthest reaches of the visible universe. Cosmic rays can have energies of over 1020 eV, far higher than the 1012 to 1013 eV that man-made particle accelerators can produce. See Ultra-high-energy cosmic rays for a description of the detection of a single particle with an energy of about 50 J, the same as a well-hit tennis ball at 42 m/s [about 94 mph]. "

Reference: Wikipedia, "Cosmic Ray."

8 Cherenkov Particle Detectors

Cherenkov detectors are very important in modern particle physics. In general the photons of the Cherenkov radiation are directed into a photo multiplier to generate a measurable electrical pulse. Dan Green has a chapter on Cherenkov radiation and detectors, with pictures of the devices.

9 Bibliography and References

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