The Corner Reflector

James Emery

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1 The Corner Reflector

Consider a ray being multiply reflected from two planes whose dihedral angle is 90 degrees. Suppose first that the ray lies a plane orthogonal to the two original planes. Let θ be the angle between the ray and the first plane it meets. Then the angle of reflection is also θ as in the figure. The reflected ray r_2 then meets the second plane, and because the two reflecting planes make an angle of 90 degrees with each other, the angle of incidence with the second plane is $\theta' = \pi/2 - \theta$, that is the complement of θ. Then as in the figure the second reflected ray must be parallel to the original incident ray, but directed oppositely.

Now suppose that the original ray is not in a plane orthogonal to the two reflecting planes. Consider the projection of the rays into a plane that is orthogonal to the two reflecting planes. Then a little thought shows that the projected rays also satisfy the law of reflection, that is the angle of incidence equals the angle of reflection. It follows as above that the final reflected ray after two reflections is parallel to the incident projected direction and directed opposite to it.

So how consider the case of three orthogonal planes forming a corner and three reflections. Without loss of generality, suppose the third plane is the x-y plane. Consider projection into the y-z plane. If a ray bounces off of the y-z plane, then the incident and the reflected ray are in a plane orthogonal to

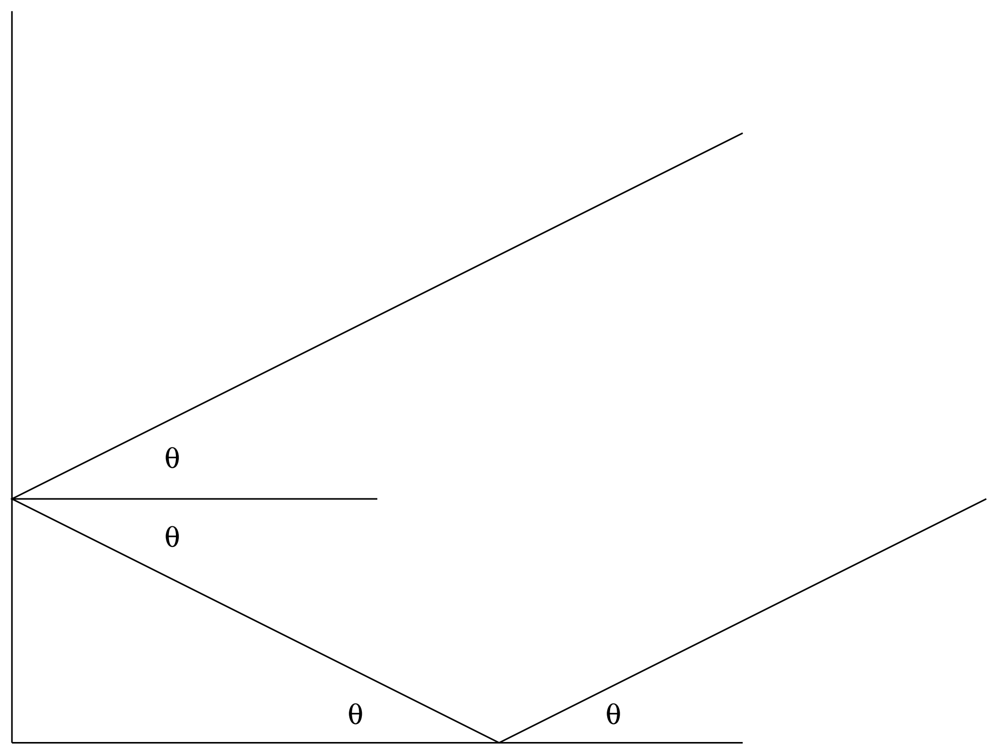

Figure 1: An incident ray bouncing off of two orthogonal planes returns parallel to the initial direction, when it is in a plane orthogonal to both reflecting planes. When it is not in a plane orthogonal to both reflecting planes, the projection of the rays into such an orthogonal plane have the same property. That is, this figure can represent a side view, of the rays when they are not necessarily in the plane of the paper.

the y-z plane. So in the projection both rays will appear as a single ray. Then the projection will look just like a projection of a double reflection involving the x-y plane and the x-z plane, and so the final ray as a projection will be directed parallel to the initial ray as a projection as above. The same result will hold for projection into the x-z plane. So in two orthogonal projections the final ray is parallel to the initial. Hence the actual final ray in three dimensions must be in the same direction as the initial ray. That is if two lines have the same direction in two orthogonal projections, then they must have the same direction in 3-dimensional space. To see this, consider that if two lines appear parallel in an orthogonal projection, then they lie in a pair of parallel planes. If they appear parallel in a second orthogonal projection then again they each lie in a second pair of parallel planes. And further each line is given by the intersection of the two planes in which it lies. Then clearly these two lines must be parallel in 3-space.

Hence a corner reflector always send a reflected ray back in the initial direction, possibly shifted parallel to itself.