Hot Wire: The Electricaly Heated Wire Experiment Showing a Phase Change in Iron

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Contents

1 Introduction

Pictures for this experiment are in the power point files with versions of the name hotwire.ppt and hotwire.pdf

2 Iron and Low Carbon Steel

Low carbon steel at low temperatures is in a body centered cubic form, which is called Ferrite. At a higher temperature, it takes a face centered cubic form, which is called Austinite. Austinite is close packed and more dense than Ferrite. Austinite can contain up to about 2 per cent carbon in solution. Ferrite can contain very little carbon in solution. The melting point of Iron is 1538 C . The density of Iron is 7.874 $g \cdot cm^{-3}$.

3 The Experiment

A 22 gauge wire is heated with 60 Hertz alternating electric current. The wire is of low carbon steel and is galvanized, that is, coated with zinc. The zinc coating soon burns off. The wire is suspended from two points that are about 1.08 meters apart. There is a small weight hanging from the middle of the wire. The middle point of the wire is 9 cm below the line joining the suspension points. This wire has been heated before. About 14 amperes pass through the wire in the beginning. In a few seconds the wire becomes bright red hot, and the current decreases to 10 amps. The wire, as it becomes red, becomes nonmagnetic. So the wire has been heated above the Curie point, which is 770C. The voltage starts low at 4 or 5 volts, then increases to about 45 volts. So the resistance starts at about .35 ohms and increases to about 4.5 ohms. The resistivity of Iron at room temperature is 60.36 ohms per circular mill. So at room temperature the resistence of an iron wire of this length and diameter should be about .33 ohms, so the starting value of the resistence seems about right. The area of a circle in circular mils is defined to be

$$A = (1000d)^2,$$

where d is the diameter in inches. The temperature coefficient of resistance for iron is .00567 ohms per degree centigrade, and for .5 per cent carbon steel is .003. Let us assume a value of $\alpha = .0045$ ohms per degree centigrade. Taking $T_1 = 25$, we have

$$(T_2 - T_1)\alpha = R_2 - R_1,$$

or

$$T_2 = T_1 + (R_2 - R_1)/\alpha = 25 + (4.5 - .35)/(.0045) = 947C$$

The wire is in series with two parallel connected electric heaters to limit the current. The weight lowers considerably as the wire expands. When the power is turned off, the wire innitially rises, but then falls a bit before rising again. It appears that the wire cools and shortens until the steel begins a phase change, changing from Austinite to Ferrite. At this point the wire lengthens because the Ferrite in BCC form is less dense then the FCC Austinite and so the wire gets longer (See below for the density ratio). Referring to the phase diagram for steel, this whould happen at a temperature somewhere between 723C and 931C. Then as cooling continues the wire shortens as the atoms in the unit cell decrease their vibration, and thus take up less space. This is the thermal contraction process.

In doing this experiment repeatedly, the response seems to become weaker, with the wire not lengthening nearly as much. The first time current is passed through the wire and it is heated for the first time, the weight lowers more than it does on the second heating. This result is probably partially due to the wire having kinks and not being perfectly straight. As the wire is heated it becomes soft and the kinks are eliminated. However, something else seems to be going on. Once the wire has been heated and cooled and so annealed, it appears to have changed from its native state.

The current was measured with a clamp meter, and the voltage with a small digital multimeter.

4 American Wire Gauge (Wikipedia)

American wire gauge (AWG), also known as the Brown and Sharpe wire gauge, is a standardized wire gauge system used since 1857 in the United

States and other countries for the diameters of round, solid, nonferrous, electrically conducting wire. The steel industry uses a different numbering system for their wire thickness gauges (for example, WandM Wire Gauge or US Steel Wire Gauge or the different Music Wire Gauge) so data below does not apply to steel wire. Since AWG is specifically for electrical conductors, the cross-sectional area of each gauge is an important factor for determining its current-carrying capacity. Increasing gauge numbers give decreasing wire diameters, which is similar to many other nonmetric gauging systems. This is derived from the fact that the gauge number is related to the number of drawing operations that must be used to produce a given gauge of wire; very fine wire (for example, 30 gauge) requires more passes through the drawing dies than does 0 gauge wire. The AWG size is one of the essential specifications that are printed on data cables. For instance, an AWG of 24 is common for network cables such as a Category 5 UTP, and an AWG of 26 is the norm for Serial ATA cables. Although the AWG tables are normally for a single, solid, round conductor, there are many cases in which AWG is applied to wires with multiple strands. When a stranded wire needs to be converted to an AWG equivalent size, the cross-sectional area of the conductor, which determines its current-carrying capacity and electrical resistance (not its diameter), is taken as the determining factor. This permits stranded wire to have a slightly different diameter than solid wire having the same AWG. AWG is also commonly used to specify body piercing jewelry sizes, especially smaller sizes.[2] Formula By definition, No. 36 AWG is 0.005 inches in diameter, and No. 0000 is 0.46 inches in diameter. The ratio of these diameters is 92, and there are 40 gauge sizes from No. 36 to No. 0000, or 39 steps. Using this common ratio, wire gauge sizes vary geometrically according to the following formula: The diameter of a No. n AWG wire in inches is

$$D_n = .005(92^{(36-n)/39})$$

Thus 22 gauge wire has diameter

$$D_n = .005(92^{14/39}) = .0253$$

inches, or .644 mm. However, the AWG is a gauge for electrical wire, and the gauge for steel wire differs a bit from this. The wire used in the experiment had a measured diameter of .69 mm.

5 FCC, BCC Density Ratio

Consider a cube of edge a. The face diagonal has length $\sqrt{2}a$. The cube diagonal has length

$$\sqrt{2a^2 + a^2} = \sqrt{3}a.$$

In the Face Centered Cubic form a unit cell has three atomic spheres touching along a face diagonal. The vertices of the unit cell are the centers of the spheres at the eight corner vertices. So the length of the face diagonal is the sum of two sphere radii and a diameter, and so equals $4r$, . Then the edge length is

$$a_1 = \frac{4r}{\sqrt{2}} = 2\sqrt{2}r.$$

The number of atoms in the unit cell is 1/8 from each of the 8 corner points, and 1/2 from each of the 6 face spheres, giving a total of 4. The atomic density for the FCC form is then

$$\rho_1 = \frac{4}{(2\sqrt{2}r)^3} = \frac{4}{16\sqrt{2}r^3} = \frac{\sqrt{2}}{8r^3}.$$

In the Body Centered Cubic form a unit cell has three atomic spheres touching along the cube diagonal. Therefore the cube diagonal has length $4r$. Then the edge length is

$$a_2 = \frac{4r}{\sqrt{3}}.$$

The number of atoms in the unit cell is 1/8 from each of the 8 corner atoms and 1 from the center atom, giving a total of 2. The atomic density is then

$$\rho_2 = \frac{2}{(\frac{4r}{\sqrt{3}})^3} = \frac{3\sqrt{3}}{32r^3}.$$

The ratio of the densities of the FCC form to the BCC form is then

$$\frac{\rho_1}{\rho_2} = \frac{4\sqrt{2}}{3\sqrt{3}} = 1.08866$$

Let us write coordinates for atoms of radius r in the FCC cell, where the cell is centered at the origin. Let one half of the edge length be

$$b = \sqrt{2}r.$$

The top vertices of the cube, in a counterclockwise order, viewed from the top, are

$$v_1 = (b, b, b)$$
$$v_2 = (-b, b, b)$$
$$v_3 = (-b, -b, b)$$
$$v_4 = (b, -b, b).$$

And the bottom vertices of the cube, in a counterclockwise order, viewed from the top, are

$$v_5 = (b, b, -b)$$
$$v_6 = (-b, b, -b)$$
$$v_7 = (-b, -b, -b)$$
$$v_8 = (b, -b, -b).$$

The face centered vertices are
Top:

$$v_9 = (v_1 + v_3)/2$$

Bottom:

$$v_{10} = (v_5 + v_7)/2$$

Front:

$$v_{11} = (v_1 + v_8)/2$$

Right:

$$v_{12} = (v_1 + v_6)/2$$

Rear:

$$v_{13} = (v_2 + v_7)/2$$

Left:

$$v_{14} = (v_3 + v_8)/2$$

We can use these vertices to make a VRML model. Programs and files to make such a model are: fcc.cpp, fccop.cpp, molecule.cpp, and, molecule.h, the later of which are molecule class definitions.

6 Estimate of Temperature by Color

Estimating temperature of steel by color is subjective, and the color temperature pairs lists do not completely agree. The tempering color estimates use the color of the oxide that forms on the surface. The following tables are the Halcomb Steel Estimates.

Color	**Degrees C**
Red heat, visible in the dark	400
Red heat, visible in the twilight	474
Red heat, visible in the daylight	525
Red heat, visible in the sunlight	581
Dark red	550-625
Dull cherry-red	800
Cherry-red	900
Bright cherry-red	1000
Orange-red	1100
Orange-yellow	1200
Yellow-white	1300
White welding heat	1400
Brilliant white	1500
Dazzling white (Bluish-white)	1600

Colors for Tempering	Degrees C
Very pale yellow	221.1
Light yellow	226.7
Pale straw-yellow	232.2
Straw-yellow	237.8
Deep straw-yellow	243.3
Dark yellow	248.9
Yellow-brown	254.4
Brown-yellow	260.0
Spotted red-brown	265.6
Brown-purple	271.1
Light purple	276.7
Full purple	282.2
Dark purple	287.8
Full blue	293.3
Dark blue	298.9
Very dark blue	315.6

7 Coefficient of Thermal Expansion

Let a wire be suspended between two points separated by distance a. Let a weight be suspended from the middle and let the middle point be located a distance b from the straight line joining the two suspension points. Then the length of the wire is

$$c = 2\sqrt{a^2/4 + b^2}.$$

The coefficient of linear expansion for steel for temperatures between 0C and 100C is (From a table in Halliday and Resnick, "Physics", 3rd ed., 1978)

$$\alpha = 11 \times 10^{-6}$$

reciprocal centegrade degrees.

$$\Delta\ell = \alpha\ell\Delta T.$$

$$\alpha = \Delta\ell/(\ell\Delta T).$$

We have $a = 1080$ mm ,$b_1 = 84.5$ mm, and $b_2 = 115$ mm. We can compute α (assuming that it is constant between say 25C and 1000C, which it probably is not). We use a Matlab or Octave script:

```
% linearexpansion.m
% a= distance between suspension points.
% b1= distance down of suspended weight at room temperature.
% b2= distance down of suspended weight at red hot temperature.
% c1= length of wire at room temperature.
% c2= length of wire at red hot temperature.
% t1= room temperature.
% t2= red hot temperature.
% alpha= linear coefficient of expansion.

a=1.08
b1=.0845
b2=.115
c1 = 2*sqrt((a/2)^2 + b1^2)
c2 = 2*sqrt((a/2)^2 + b2^2)
t1=25
t2=925
dt=t2-t1
dc=c2-c1
alpha = dc/(a* dt)
```

We get the following numbers:

```
a =  1.0800
b1 =  0.084500
b2 =  0.11500
c1 =  1.0931
c2 =  1.1042
t1 =  25
t2 =  925
dt =  900
dc =  0.011076
```

```
alpha = 1.1396e-005
```

The value calculated for $\alpha, 11.396 \times 10^{-6}$, is surprisingly close to the table value of 11×10^{-6}.

8 Maxwell's Equations

Recall that if

$$\mathbf{C} = C_x\mathbf{i} + C_y\mathbf{j} + C_z\mathbf{k},$$

then

$$\nabla \times \mathbf{C} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \\ C_x & C_y & C_z \end{vmatrix}$$

and

$$\nabla \cdot \mathbf{C} = \frac{\partial C_x}{\partial x} + \frac{\partial C_y}{\partial y} + \frac{\partial C_z}{\partial z}.$$

The Maxwell Equations in MKS form are

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t},$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \cdot \mathbf{D} = \rho,$$

$$\nabla \cdot \mathbf{B} = 0.$$

The first equation is Ampere's Law with the Maxwellian addition of the displacement current term

$$\frac{\partial \mathbf{D}}{\partial t}.$$

If there is no changing field $\mathbf{D}$, which is a modification of the $\mathbf{E}$ field caused by the presence of electrically polarized materials, then the first equation becomes

$$\nabla \times \mathbf{H} = \mathbf{J}.$$

The vector field $\mathbf{D}$ is defined by

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P},$$

where $\mathbf{P}$ is the electric dipole moment per unit volume in a dielectric material. The number ϵ_0 is called the permitivity of free space.

Applying Stoke's Theorem we have

$$\int_C \mathbf{H} \cdot d\mathbf{R} = \int_S \nabla \times \mathbf{H} \cdot d\mathbf{S} = \int_S \mathbf{J} \cdot d\mathbf{S} = i.$$

That is, the line integral of the magnetic intensity $\mathbf{H}$ around a path C equals the amount of curremt i flowing through the surface S that is bounded by C, which is Ampere's law. Let us remark about the displacement term. Now if there were a capacitor placed in our wire, there is current flowing through the wire, but no actual charge flow between the capacitor plates. Hence, if we let our surface S pass between the capacitor plates then there would be a zero J, and thus zero i. And so our line integral of $\mathbf{H}$ around the magnetic circuit would be zero. So depending on where we place our surface we get zero or not zero. That is why the displacement term must be added to the first maxwell equation.

The second equation is Faraday's law of induction. Using Stoke's theorem we have

$$\int_C \mathbf{E} \cdot d\mathbf{R} = \int_S \nabla \times \mathbf{E} \cdot d\mathbf{S} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} = -\frac{\partial \Phi}{\partial t}.$$

That is, the electric potential (MMF) around a circuit C is equal to the rate of magnetic flux change through the circuit.

If the material is soft iron and essentially linear with little hystereses we may write

$$\mathbf{B} = \mu \mathbf{H},$$

where μ is a constant called the permeability. Such material forms a linear magnetic circuit. From the fourth law,

$$\nabla \cdot \mathbf{B} = 0.$$

The divergence of the $\mathbf{B}$ field is zero. Using the divergence theorem to convert a volume integral to a an integral on the bounding surface, we have

$$0 = \int_V \nabla \cdot \mathbf{B} dV = \int_S \mathbf{B} \cdot d\mathbf{S},$$

which means that every flux line entering a volume, leaves the volume. Thus there are no sources of magnetic flux lines and so they form continuous loops of flux.

9 The Clamp Meter

The clamp meter for measuring current is an annular ring of soft iron that closes around the wire. The clamp meter measures the alternating current in the wire. The line integral of the H field along a path located in the soft iron is equal to the amount of current that passes through the ring of soft iron. This defines a B field along each path, and hence the total flux in the magnetic circuit. This magnetic flux is proportional to the current i. A coil is wound around the magnetic path and the induced voltage is given by Faraday's law as the negative derivative of the flux with respect to time, times the number of turns in the coil. Thus the induced voltage in the coil is proportional to the current in the wire. Alternating current is sinusoidal, as is the derivative of the flux induced by the current. So the induced voltage in the sensing coil is also sinusoidal.

10 Magnetic Circuits

A continuous tube of flux Φ forms a magnetic circuit. Let the circuit pass through a coil containing N turns and current i. For a path around the circuit

$$Ni = \oint \mathbf{H} \cdot d\mathbf{r} = \sum \; H_i L_i = \sum \frac{L_i \phi}{\mu_i A_i} = \Phi \sum \Re_i.$$

This equation is an approximation. $\mathbf{H}_i$ is an assumed constant value of $\mathbf{H}$ in the ith piece of the circuit. The reluctance of the ith piece is $\Re_i$. L is the length of the piece and A is the cross sectional area. The magnetomotive force mmf is Ni. We have

$$mmf = \Phi \Re.$$

A large flux path may be treated as a set of parallel paths and the net reluctance computed in the same way of computing parallel resistances.

11 You Tube Video

http://www.youtube.com/watch?v=3lXFbU6ZhHE

12 Programs and Files

```
fcc.wrl face centered cubic vrml file generated with fcc.cpp and vrml conversion programs
bcc.wrl body centered cubic vrml file generated with bcc.cpp
fccop.wrl vrml file showing close packed planes generated with fccop.cpp
power point file: hotwire2.ppt
video of experiment using video camera on video tape, dvd copy recorded, youtube video
has been removed?
VRML Viewer used: flux player
```

13 Bibliography

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