Lissajous Figures

James Emery

4/13/2008)

Contents

1 Curves

Let

$$x(t) = a_1 \sin(\omega_1 t - \phi_1)$$
$$y(t) = a_2 \sin(\omega_2 t - \phi_2).$$

Let us compute the period of this curve, assuming that ω_1 and ω_2 are in a ratio of whole numbers. The period of x is

$$T_1 = \frac{2\pi}{\omega_1}.$$

The period of y is

$$T_2 = \frac{2\pi}{\omega_2}.$$

Therefore the curve period is

$$T = n_1 T_1 = n_2 T_2,$$

where n_1 and n_2 are integers, and are the smallest such integers. Suppose

$$\frac{\omega_1}{\omega_2} = \frac{k_1}{k_2},$$

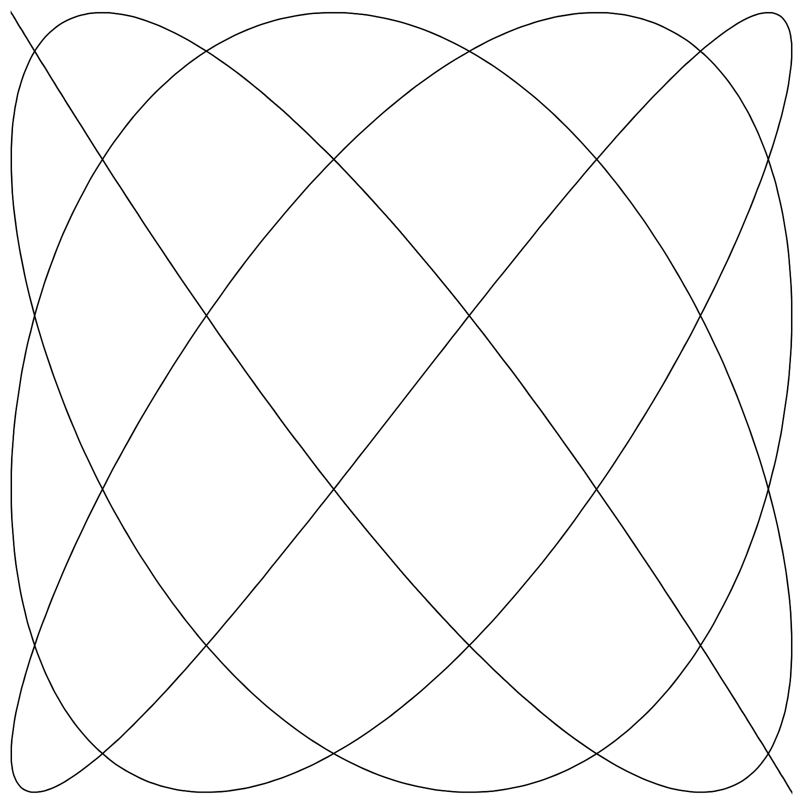

Figure 1: Lissajous figure with $k_1 = 7$ and $k_2 = 9$.

where k_1 and k_2 are relatively prime (have no common divisor). Then

$$\frac{T_2}{T_1} = \frac{\omega_1}{\omega_2} = \frac{k_1}{k_2}.$$

So shall find that we can let

$$n_1 = k_2$$

and

$$n_2 = k_1.$$

In Figure 1, the curve appears to have a start point and an end point end. But these points are actually where the curve velocity goes to zero. Then the curve reverses direction to retrace its path.

In figure 2, the curve never has a zero velocity. This behavior depends upon the values of k_1 and k_2.

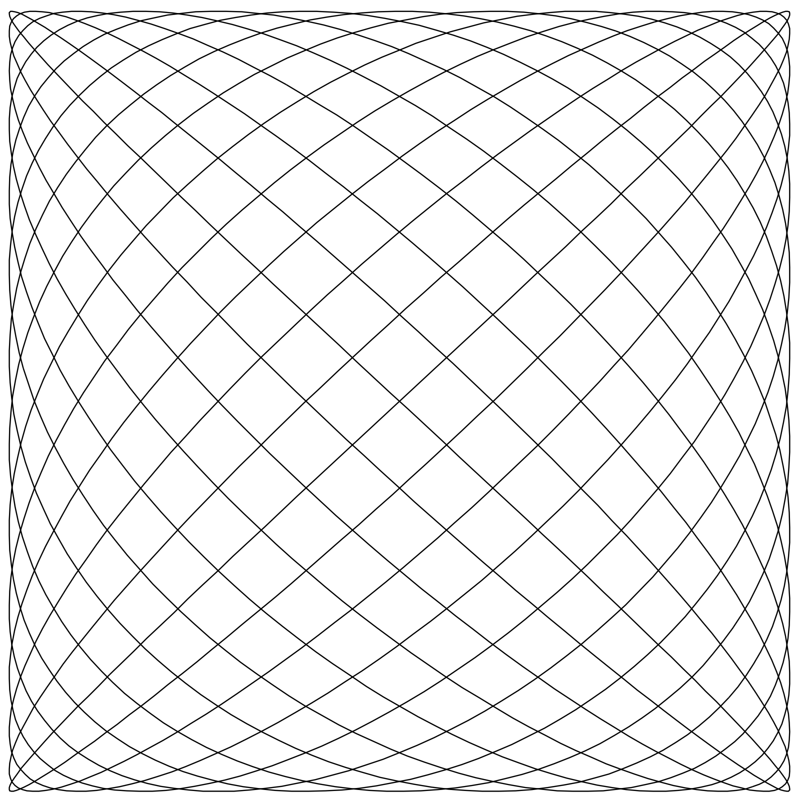

Figure 2: Lissajous figure with $k_1 = 14$ and $k_2 = 13$.

We find that the period of the curve is

$$T = n_1 T_1 = n_2 T_2.$$

This is true because k_1 and k_2 are relatively prime. Because if the period were $T' < T$, then k_1 and k_2 would have a common factor.

2 Program

Here is a computer program for making plots of Lissajous figures. We operate the program by having the source in an editor such as notepad. We specify the type of curve by changing the integers k_1 and k_2. Then we save the file. We have the postscript program Ghostview running in a window, and we have a batch file called lj.bat in a directory containing the source. In the open command line window we type "lj". This compiles the program, runs the program, and calls some utility programs to create the postscript files from the eg files. When Ghostscript sees a new version of the postscript file, it loads and displays it, which is very nice because it avoids the tedious clicking of the file open command.

Here is the batch file:

```
fl32 lissajous.ftn
lissajous
pltmerge lj.eg p.eg
eg2ps p.eg p.ps
pltmerge ljxy.eg pxy.eg
eg2ps pxy.eg pxy.ps
pltmerge ljs.eg ps.eg -w
pltax ps.eg vm.eg T Speed "Velocity Magnitude"
eg2ps vm.eg vm.ps
```

Here is what the commands do. **fl32** is the Microsoft Fortran Power Station compiler. **lissajous.exe** is the executable file produced by **fl32**. Running it produces graphic files with an **eg** extension. **lj.eg** is the Lissajous curve. **ljs.eg** is a plot of the curve velocity magnitude. The program **pltmerge** can merge several "eg" type files into a single file. To do this it needs to compute a bounding rectangle that encloses the curves. The **lissajous.ftn** program writes the graphic files without calculating a bounding window. So the purpose of **pltmerge** here is to calculate this bounding window for a single file and add an appropriate window and viewport command to the top of the output file. **pltax** adds plot axes and labels to a plot. **eg2ps** is my C program that converts an ”eg” file to a postscript file. Here is the Fortran program that writes three files of plot data:

```
c lissajous figures 4/4/08
      implicit real*8 (a-h,o-z)
      nfile=1
      open(nfile,file='lj.eg',status='unknown')
      open(2,file='ljxy.eg',status='unknown')
c     open(3,file='ljx.eg',status='unknown')
c     open(4,file='ljy.eg',status='unknown')
      open(5,file='ljs.eg',status='unknown')
      pi=3.14159265358979
      n=2000
      t1=0
      t2=10.
c     specify the ratios of the two frequencies using two relatively
c     prime numbers k1 and k2
      k1=7
      k2=9
      a1=1.
      omega1=2.*pi
      phi1=0.
c
      a2=1.
      omega2=k2*omega1/k1
      phi2=0.

       t1=0.
       t2=k1
c      t2=2.
c      k1/k2=T2/T1=omega1/omega2
c      k1 and k2 are integers and k1 is the smallest such integer
c
      do i=1,n
       t=(i-1)*(t2-t1)/(n-1)+t1
       x=a1*sin(omega1*t - phi1)
       y=a2*sin(omega2*t - phi2)
c       compute the magnitude of the velocity
       xp=a1*omega1*cos(omega1*t - phi1)
       yp=a2*omega2*cos(omega2*t - phi1)
       s=sqrt(xp*xp+yp*yp)
       if(i .eq. 1)then
        write(1,'(1x,a,2(1x,g15.8))')'m',x,y
        write(2,'(1x,a,2(1x,g15.8))')'m',t,x
        write(5,'(1x,a,2(1x,g15.8))')'m',t,s

       else
        write(1,'(1x,a,2(1x,g15.8))')'d',x,y
        write(2,'(1x,a,2(1x,g15.8))')'d',t,x
        write(5,'(1x,a,2(1x,g15.8))')'d',t,s
       end if
      enddo


      do i=1,n
```

```
 t=(i-1)*(t2-t1)/(n-1)+t1
 x=a1*sin(omega1*t - phi1)
 y=a2*sin(omega2*t - phi2)
 if(i .eq. 1)then


  write(2,'(1x,a,2(1x,g15.8))')'m',t,y
 else


  write(2,'(1x,a,2(1x,g15.8))')'d',t,y
 end if
enddo

end
```