

Octave

James Emery

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1 Introduction

Octave is a free GNU program for doing numerical mathematics whose commands are mostly compatible with **Matlab**. So much of a Matlab book can

be used to learn Octave.

2 Downloading Octave

To download GNU Octave, Google Octave. You will then select

```
http://www.gnuorg/software/octave
Then select download.
You will go to:
http://www.gnuorg/software/octave/download.html
Scroll down to "Windows" and "Octave Forge," which select.
You will go to
http://octave.sourceforge.net/
Select "Windows Installer" (or "Octave.app for MacOS X" for macintosh)
Note the location of the saved file, which for Windows is called "Octave-3.0.0-setup.exe"
Click the file to run it. Take all defaults. It will be installed. To run it select
Octave in the program menu, then run it or browse the manual in the html or pdf versions (616 pages).
```

3 A Few Command Examples

```
x = -10:0.1:10;
plot (x, sin (x));
lambda=eig(a)
[v,lambda]=eig(a)
diary % diary turns on a log of the commands issued in the session, and stores it in a file called diary
x=[1,2,3,4,5,6,7,8,9]
y=exp(x)
cd c:\je\m
testOctave1
who % lists the variables that are currently defined
pwd % (a comment is indicated by a percent sign) pwd list the current working directory
cd \je\octave % changes the working directory to c:\je\Octave
global x % make a variable global so that it can be accessed inside a function
hold on % add the next plot to the current axis
writetofile % calls the script writetofile.m located in the working directory.
quit % ends the session
```

4 Using Octave as a Calculator

Octave and Matlab can be used like a graphing calculator for doing text-book like scientific problems. For example consider finding the current in an alternating current network by combining impedances using complex numbers. Use the argument function to find phase angles. Change the frequency by backing up with the up arrow. Calculate the angle between the voltage

and current phasers to calculate power factor. Turn on diary to record your calculation.

5 Scripts

A script is a file (an m file, with extension .m) containing Octave commands. When the name of the file is typed at the Octave command line, Octave fetches the file and executes all of the commands in the file just as if they were typed at the Octave command line. For example, here is a script called "writetofile.m" that illustrates the writing of data to files in a couple of ways.

```
x = 0:.1:1;
y = [x; exp(x)]; % the ending semicolon suppresses output
y % this displays the value of the matrix y
fid = fopen('c:\txt\exp1.txt','w');
fprintf(fid,'%6.2f %12.8f\n',y);
fclose(fid);
%
y=y';
fid = fopen('c:\txt\exp2.txt','w');
for i = 1:11
    fprintf(fid,'%6.2f %12.8f\n',y(i,1),y(i,2));
end;
fclose(fid);
```

This was actually a script written for Matlab. It works in Octave, as most basic Matlab commands do. Typing the m-file name, without the m extension, such as by typing "write2file," creates two files in directory c:/txt

Here is a listing of, "/c:/txt/exp1.txt," which is one of the files produced by the script:

```
0.00    1.00000000
0.10    1.10517092
0.20    1.22140276
0.30    1.34985881
0.40    1.49182470
0.50    1.64872127
0.60    1.82211880
```

```
0.70  2.01375271
0.80  2.22554093
0.90  2.45960311
1.00  2.71828183
```

The file is written with the Unix end of line characters, namely linefeeds (lf). So to convert to the windows end of line characters "crlf," one may use a utility program, like say my C program "lf2nl.c".

6 Linear Equation Solution

Consider the equations

$$2x + 3y + 4z = 9$$

$$x - y - z = -1$$

$$5x + y - z = 5$$

We can solve these using

```
a=[2,3,4;1,-1,-1;5,1,-1]
b=[9;-1;5]
ai=inverse(a)
c=ai*b
```

Now we computed the inverse of matrix a and then multiplied. This is an expensive calculation, because a solution to a linear set of equations can be solved without computing the inverse and multiplying. So the more efficient solution method is available in Matlab or Octave by using the backwards division operator:

```
c = a\b
```

7 Fourier Transform Example

The Fourier transform of the function f is defined as (Goldberg, **The Fourier Transform**)

$$g(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t)e^{-i\omega t} dt.$$

By the Fourier integral theorem

$$f(t) = \int_{-\infty}^{\infty} g(\omega)e^{i\omega t} d\omega.$$

Example. Let

$$f(t) = \begin{cases} 2e^{-3t} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Then

$$g(\omega) = \frac{2}{3 + i\omega}$$

Also see foran.tex

Here is a script file to compute the fft in matlab or octave:

```
%Example page 229, Mastering Matlab 5, fft
N=128;
t=linspace(0,3,N);
f=2*exp(-3*t);
Ts = t(2) - t(1);
Ws = 2*pi/Ts;
F=fft(f);
Fp=F(1:N/2+1)*Ts;
W=Ws*(0:N/2)/N;
Fa=2./(3+j*W);
plot(W,abs(Fa),W,abs(Fp),'+')
xlabel('Frequency, Rad/s')
ylabel('|F(\omega)|')
title('Figure 21.1; Fourier Transform Approximation')
```

8 Reading Data From a File as a Matrix

A data file consisting of numbers, with say m records, each consisting of n numbers, can be read into Octave or Matlab with the script `rdmat.m`, which is listed here. The numbers on a row, may be separated with blanks, or with a comma. So if data is saved from Excel as a comma delimited list, then it may be read into Octave as a matrix. So for example suppose one has some

xy data in a file called a.dat in the Octave working directory (Remember, one can type "pwd" to print your working directory, and can change the working directory using "cd"). To plot this data one might do this:

```
a=rdmat('a.dat')
x=a(:,1)
y=a(:,2)
plot(x,y)
```

Or perhaps the coefficients of the left side and right side of a linear equation are stored in a.dat, and we wish to read in the data and solve the linear equation.

```
m=rdmat('a.dat')
a=m(1:3,1:3)
b=m(1:3,4)
x=a\b % solution of a*x = b
```

Here is the script rdmat.m:

```
%read a matrix from a file
% Example usage: m=rdmat('c:\je\octave\test.txt')
function a=rdmat(fname)
    fid=fopen(fname,'r');
    s1=fgets(fid);
    k=1;
    while(!feof(fid))
        if k == 1
            s=s1;
        else
            s=[s;s1];
        end
        s1=fgets(fid);
        k=k+1;
    end
    a=str2num(s);
endfunction
```

9 Solving a Differential Equation Numerically

To solve a differential equation numerically, one must first write it as a system of first order equations, and specify the initial conditions. Here we call lsnode to solve such an equation.

```
%solve differential equation from octave manual,
%where fde is defined in file fde.m
```

```

x0 = [1; 2]; % set initial conditions
% select 200 equally spaced time points between 0 and 50
t = linspace(0, 50, 200);
x = lsode ("fde", x0, t);
plot (t, x)

```

The right hand side of the differential equation is defined in a function script called fde.m. Note that the name of the function and the file name must agree.

```

function xdot = fde (x, t)
    r = 0.25;
    k = 1.4;
    a = 1.5;
    b = 0.16;
    c = 0.9;
    d = 0.8;
    xdot(1) = r*x(1)*(1 - x(1)/k) - a*x(1)*x(2)/(1 + b*x(1));
    xdot(2) = c*a*x(1)*x(2)/(1 + b*x(1)) - d*x(2);
endfunction

```

10 Damped One-Dimensional Vibration

The differential equation for damped vibration is

$$m\ddot{x} + c\dot{x} + kx = F(t).$$

The mass is m , the viscous damping constant is c , the stiffness is k , and the applied force is $F(t)$. For free damped vibration the roots of the characteristic equation are

$$-\frac{c}{2m} \pm \sqrt{(c/2m)^2 - k/m}.$$

The natural undamped resonant angular frequency is

$$\omega_n = \sqrt{k/m}.$$

The critical damping constant is

$$c_c = 2m\omega_n.$$

The critical damping ratio is defined to be

$$\zeta = \frac{c}{c_c}.$$

We have

$$c = 2\zeta m\omega_n,$$

so that if we divide the damped vibration equation by the mass m , we can write it in the form

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2x = \frac{F(t)}{m}.$$

Let us suppose that the force is zero $\zeta < 1$ so that we have a case of decaying oscillation. The equation becomes

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2x = 0.$$

Solving this equation by substituting in $\exp(pt)$ to get the characteristic polynomial in p , we find that

$$x = x_0 \exp(-\zeta\omega_n t) \sin(\omega_d t + \phi),$$

where the angular frequency of damped oscillation is

$$\omega_d = \omega_n \sqrt{1 - \zeta^2},$$

and the corresponding period is

$$\tau_d = \frac{2\pi}{\omega_d}.$$

The notation in this section follows *Thomson: Theory of Vibration With Applications* Let us consider the case where $x(0) = 0$, so that the phase factor ϕ is zero, we have

$$x = x_0 \exp(-\zeta\omega_n t) \sin(\omega_d t).$$

Let us take the natural frequency to be

$$\omega_n = 2\pi,$$

so that the natural period is

$$T_n = 1.$$

Suppose $\zeta = 1/2$, so that ω_d is about 5.4414, and the damped period is about

$$T_d = \frac{2\pi}{\omega_d} = 1.1547\text{sec.}$$

Also suppose $x_0 = 1$. Here is an octave script called dampedh.m:

```
zeta=1/16;
omega_n=2*pi;
omega_d=omega_n*sqrt(1.-zeta^2);
n=200;
t1=0.;
t2=3.;
t=linspace(t1,t2,n);
for k= 1:n
    x(k)=exp(-zeta*omega_n*t(k))*sin(omega_d*t(k));
end
plot(t,x)
```

Now let us compute this solution numerically. We want to solve

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2x = 0.$$

Given the value of x and its first derivative (\dot{x}) at 0, there is a unique solution. The numerical solution should agree closely with

$$x(t) = \exp(-\zeta\omega_n t)\sin(\omega_d t).$$

To solve a second order equation numerically, we must convert it to a first order system. So define

$$\begin{aligned} u_1 &= x, \\ u_2 &= \frac{dx}{dt}. \end{aligned}$$

So we get the system

$$\begin{aligned} \frac{u_1}{dt} &= u_2, \\ \frac{u_2}{dt} &= \frac{d^2x}{dt^2} = -(2\zeta\omega_n u_2 + \omega_n^2 u_1). \end{aligned}$$

For the closed form solution computed above, which we want the numerical solution to match, we have

$$x(0) = 0,$$

and the derivative of x is

$$\frac{dx}{dt} = -\zeta\omega_n \exp(-\zeta\omega_n t) \sin(\omega_d t) + \exp(-\zeta\omega_n t)\omega_d \sin(\omega_d t).$$

So

$$\frac{dx(0)}{dt} = \omega_d.$$

So the required initial conditions for our system are

$$u_1(0) = 0,$$

and

$$u_2(0) = \omega_d.$$

A script, called `dampedhn.m`, for solving the equation numerically, is

```
% dampedhn.m solving a differential equation
% for damped harmonic vibration numerically.
% requires script fdedh.m for the left side of the equation
% See octave.tex
zeta=1/16;
omega_n=2*pi;
omega_d=omega_n*sqrt(1.-zeta^2);
n=200;
t1=0.;
t2=3.;
% set the initial conditions
u0 = [0; omega_d];
% select n equally spaced time points between t1 and t2
t=linspace(t1,t2,n);
u = lsode ("fdedh", u0, t);
```

Before we can run this script we have to provide a definition of the left side of our DE system in the function "fdedh," in an m file with the same name.

```

%fdedh.m leftside of a differential equation for damped harmonic motion
%called by dampedhn.m
function udot = fdedh (u, t)
    zeta=1/16;
    omega_n=2*pi;
    udot(1) = u(2);
    udot(2) = -(2*zeta*omega_n*u(2) + omega_n^2*u(1));
endfunction

```

To compare the values from the closed form solution and the numerical solution we may put them in a two column matrix and list it.

```

x1=x'; %convert x to a column vector
u1=u(:,1) % select the first column of u
for k=1:200 % put x1 and u1 into a 2 column matrix
    m(k,1)=x1(k)
    m(k,2)=u1(k)
end
m % list m

```

11 Bibliography

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- [3]Hanselman Duane, and Littlefield Bruce, **Mastering Matlab 5, A Comprehensive Tutorial and Reference**, Prentice-Hall, 1998.
- [4]Eaton John W., Bateman David, Hauberg Soren, **GNU Octave: A high-level interactive language for numerical computations**, Edition 3 for Octave version 3.0.0, July 2007 (PDF document).