Using the 3D Pantograph for Scanning a Surface.

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Refer to figure 1. Let

$$A = (0, 0)$$

$$B = d_{AB}(\cos(\phi), \sin(\phi),$$

where d_{AB} is the distance from point A to B. Let D lie on a line from A to B so that

$$D = d_{AD}(\cos(\phi_1), \sin(\phi_1)$$

Define

$$F = d_{AF}(\cos(\theta), \sin(\theta)$$

Let C be chosen on the line through A and F, so that line segment BC is parallel to line segment DF. Then let E be a point on the line through D

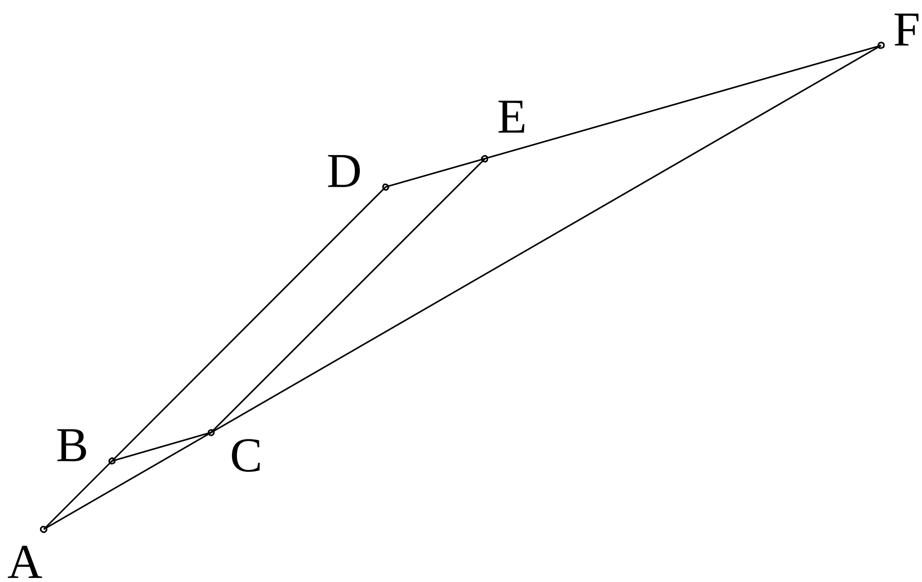

Figure 1: The pantograph scales a figure traced out at C to an enlarged figure, with scale factor α, traced out at F. A is a fixed point. $BCED$ is a parallelogram. The links pivot at the joints B, C, D and E. Scale factor α is equal to the ratio $AD/AB = AF/AC$. Points A, C, and F are collinear points, but lines AC and CF do not represent physical linkages, and their lengths vary. ABC and ADF are maintained as similar triangles by the parallel linkages..

and F so that segments BD and CE are parallel. All of this simply means that $BCED$ is a parallelogram. We see that triangles ABC and ADF are similar. It follows that for any value of θ

$$\frac{d_{AF}}{d_{AC}} = \frac{d_{AD}}{d_{AB}}$$

Let α be the constant

$$\alpha = \frac{d_{AD}}{d_{AB}}.$$

If

$$C = (x_C, y_C)$$

and

$$F = (x_F, y_F)$$

we have

$$F = \alpha C$$

So if point C traces out a figure, then F traces out a similar figure enlarged by the scale factor α. Here is a Python program to define the points A, B, C, D, E, F and draw the figure with node definitions for program **cdiagram.ftn**.

```
# pantograph figure, program pantograph.py
import math
ax=0.
ay=0.
phi=math.pi/4.
dab=1.
bx=dab*math.cos(phi)
by=dab*math.sin(phi)
print "n ",1,ax,ay
print "n ",2,bx,by
dad=5.
dx=dad*math.cos(phi)
dy=dad*math.sin(phi)
print "n ",3,dx,dy
theta=math.pi/6.
daf=10.
fx=daf*math.cos(theta)
fy=daf*math.sin(theta)
print "n ",4,fx,fy
alpha = 5.
dac=daf/alpha
cx=dac*math.cos(theta)
cy=dac*math.sin(theta)
print "n ",5,cx,cy
```

```
ufx=fx-dx
ufy=fy-dy
ex=dx+ufx/alpha
ey=dy+ufy/alpha
print "n ",6,ex,ey
print "w 1 3"
print "w 1 4"
print "w 3 4"
print "w 2 5"
print "w 5 6"
print "a 1 -l   A"
print "a  2 -l   B"
print "a  3 -l D"
print "a  4 -l F"
print "a  5 -l C"
print "a  6 -l E"
print "con 1 "
print "con 2 "
print "con  3 "
print "con  4 "
print "con  5 "
print "con  6 "
```

3 A 3d Pantograph

A 3d pantograph is used in creating molds for sculptures. If the pivot of the planar pantograph is a ball joint, then the pantograph mechanism produces a 3d scaling because certainly if the paralellograms of the linkage remain fixed and the whole thing is just pivoted on the ball joint, then the scale factor between the two pointers is preserved. It is also interesting to see that if we measure the angle between line AD and AF we can calculate the distance d_{AF}. Then if we measure the polar angle of AF and the azmuthal angle of AF, we have the spherical coordinates of point F. Thus we can use this device for mechanically scanning the surface of a 3d object. We can get to the back of the object if we are allowed to rotate the object knowing the angle of rotation.