

Piezoelectricity

Jim Emery

4/3/97

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0.1 Introduction

We shall show how to derive the stress form of the piezoelectric equations from the strain form, and we shall show how to find the clamped permittivity matrix from the unclamped matrix. These conversions are required for finite element programs because in such programs equations are solved for the elastic displacements, and so the strains rather than the stresses are the independent variables. We shall do the basic derivation three times in the following sections. We shall first treat the one dimensional equation. In later sections we shall treat the matrix case and the full tensor case. These derivations are conceptually the same.

0.2 The Linear Piezoelectric Equations

A piezoelectric ceramic is a ferroelectric material. It exhibits hysteresis and has nonlinear behavior. When this material is poled (placed in a strong electric field), it attains a permanently polarized state. For a small variation in the electric field, it behaves approximately linearly near that state.

The linear piezoelectric equations are written in two different forms. In the more common form, which is called the strain form, the strain tensor ε_{ij} and the electric polarization vector P_i are each written as linear functions of the electric field vector E_i and the stress tensor σ_{ij} . These equations are

$$\varepsilon_{ij} = E_k d_{kij} + c'_{ijkl} \sigma_{kl},$$

and

$$P_i = \epsilon_0 \chi_{ij} E_j + d_{ijk} \sigma_{jk}.$$

The d_{ijk} are the components of the piezoelectric tensor (strain coefficients). The c'_{ijkl} are the components of the inverse elastic tensor. The constant ϵ_0 is the permittivity of free space. The χ_{ij} are the components of the electric susceptibility tensor. In the direct piezoelectric effect, when a piezoelectric material is put under a stress, the material becomes electrically polarized and surface charges appear. The direct piezoelectric effect is

$$P_i = d_{ijk} \sigma_{jk},$$

which is obtained from the second equation, when the external electric field is zero.

In the converse piezoelectric effect, when a piezoelectric material is put in an external electric field (voltages applied to electrodes), the material experiences a strain. The converse piezoelectric effect is

$$\varepsilon_{ij} = E_k d_{kij},$$

which is obtained from the first equation, when there are no external forces and the stress is zero.

The fact that the coefficients for both the direct effect and the converse effect are the same is a consequence of conservation of energy. This fact can be established by a thermodynamic argument (See Nye). Because of various symmetries in the tensor indices, the equations have matrix forms. Let us define an index function $f : (i, j) \rightarrow k$ that takes a pair of indices and produces a single index. The function f takes values as follows: $f(1, 1) = 1, f(2, 2) = 2, f(3, 3) = 3, f(2, 3) = 4, f(1, 3) = 5, f(1, 2) = 6$. The function is symmetric, $f(i, j) = f(j, i)$, and so $f(3, 2) = 4, f(3, 1) = 5, f(2, 1) = 6$. This is the Physics and The Institute of Radio Engineers assignment function. We define this function with a table:

f	1	2	3
1	1	6	5
2	6	2	4
3	5	4	3

This index assignment is the one most often used in elasticity theory to convert the second rank stress and strain tensors to six component vectors. Although this is the usual convention, the following assignment defined by function g is used sometimes (ABAQUS).

g	1	2	3
1	1	4	5
2	4	2	6
3	5	6	3

A third index function is also used. The function, which we call h is used by the finite element program ANSYS.

h	1	2	3
1	1	4	6
2	4	2	5
3	6	5	3

Warning! These differences in index assignment will change the matrices such as c, d, e .

The matrix components of the piezoelectric tensor are $d_{i,f(i,j)}$ and are either equal to the tensor component itself, when $i = j$, or to twice the component. This will be explained below.

There is a second form of the piezoelectric equations, which is called the stress form.

The stress tensor σ_{ij} and the electric displacement vector D_i , are each written as linear functions of the electric field vector E_i and the strain tensor ε_{ij} . These equations are

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} - E_k e_{kij},$$

and

$$D_i = e_{ijk}\varepsilon_{jk} + \epsilon_{ij}E_j.$$

The c_{ijkl} are the components of the elastic tensor. The e_{ijk} are the components of the piezoelectric tensor (stress coefficients). The ϵ_{ij} are the components of the permittivity tensor.

These equations are suitable for finite element programs because in these programs the displacements and hence the strains are the independent variables, whereas the boundary conditions are given as force loadings, that is as stresses. It will be shown below that the piezoelectric stress coefficients and the strain coefficients are related as follows:

$$e_{ijk} = c_{jklm}d_{ilm}.$$

Notation varies greatly in the field of piezoelectricity. In a following section we will summarize the notation used in physics, and in two finite element programs ABAQUS and ANSYS.

0.3 Piezoelectric Ceramics Constants and One Dimensional Equations

The principal references for this section are: Jaffe, Cook and Jaffe, **Piezoelectric Ceramics**, and the Morgan Matroc document **Guide To Modern Piezoelectric Ceramics**.

The original research on piezoelectric materials was conducted with parallel plate capacitors, and the problem was treated as one dimensional. So the strain form of the equations considered were

$$\varepsilon = Ed + c^{-1}\sigma$$

$$P = \epsilon_0\chi E + d\sigma,$$

where the quantities are in the 3 direction. From the definition of the electric displacement

$$D = P + \epsilon_0 E = \epsilon_0(\chi + 1)E + d\sigma = \epsilon^\sigma E + d\sigma.$$

So we have an alternate form

$$D = \epsilon^\sigma E + d\sigma.$$

Here ϵ^σ is the permittivity at constant stress (unclamped).

The second set of piezoelectric equations (stress form) may be obtained from the first set (strain form). Solving the first equation for the stress, we have

$$\sigma = c\varepsilon - cdE = c\varepsilon - eE.$$

Making this substitution for σ in the alternate form of the second equation of the first set, we get the second equation of the second set.

$$D = \epsilon^\sigma E + d(c\varepsilon - eE) = \epsilon E + e\varepsilon,$$

$$= (\epsilon^\sigma - de)E + dc\varepsilon$$

$$= \epsilon^\varepsilon E + e\varepsilon,$$

where the piezoelectric coefficient $e = cd$. Here ϵ^ε is the permittivity at constant stress (unclamped).

The coefficients may be defined as partial derivatives. For example,

$$d = \left(\frac{\partial \varepsilon}{\partial E}\right)_\sigma,$$

where the subscript σ means that ε is a function of E and σ , where σ is held constant in computing the partial derivative. Similarly we have

$$d = \left(\frac{\partial P}{\partial \sigma}\right)_E$$

The constant g is defined by

$$g = \left(-\frac{\partial E}{\partial \sigma}\right)_\varepsilon = \left(\frac{\partial \varepsilon}{\partial D}\right)_\sigma.$$

We have

$$E = \frac{1}{\epsilon}(D - d\sigma)$$

so

$$g = -\left(\frac{\partial E}{\partial \sigma}\right)_\varepsilon = \frac{d}{\epsilon}.$$

The constant e is defined by

$$e = -\left(\frac{\partial \sigma}{\partial E}\right)_\varepsilon = \left(\frac{\partial D}{\partial \varepsilon}\right)_E$$

The poled ceramic is an orthotropic material with a plane of symmetry whose normal is in the poled direction. Further it is symmetric with respect to any rotation about the poling direction. Performing a reflection through the symmetry plane, some of the strain components and stress components are maintained in sign and some others are changed in sign. Writing the various components of stress as functions of strain, in the two coordinate systems, we find that the proper signs can be maintained only if some of the elastic coefficients are zero (see pp62-64 Sokolnikoff **Mathematical Theory of Elasticity**). Carrying out rotation transformations about the poling axis (3 direction) we find further conditions on the elasticity coefficients. We have only the following independent elastic coefficients (Jaffe et al, p20)

$$c_{11}, c_{33}, c_{44}, c_{12}, c_{13}.$$

We have

$$c_{22} = c_{11}, c_{55} = c_{44}, c_{66} = 2(c_{11} - c_{12}), c_{23} = c_{13}.$$

Coefficients which are not in the upper 3-dimensional submatrix or on the main diagonal are zero.

The independent piezoelectric coefficients are

$$d_{33}, d_{31}, d_{15}.$$

We have

$$d_{32} = d_{31}, d_{24} = d_{15}.$$

All other coefficients are zero.

The independent dielectric coefficients are

$$K_{11}, K_{33}$$

We have

$$K_{22} = K_{11}.$$

All other coefficients are zero.

The permittivity is

$$\epsilon_{ij} = \epsilon_0 K_{ij}.$$

The electromechanical coupling constant is defined in terms of the ratio of stored electrical to stored mechanical energy. Let Q_{mech} be the electrical energy converted to mechanical energy and let Q_{input} be the electrical energy input. Then the electromechanical coupling factor is

$$k^2 = \frac{Q_{mech}}{Q_{input}}.$$

It is claimed (Jaffe et al p7) that the relation between the unclamped dielectric constant K^σ and the clamped dielectric constant K^ϵ is

$$K^\epsilon = K^\sigma(1 - k^2).$$

Because $0 < k < 1$, the clamped dielectric constant is smaller than the unclamped constant

$$K^\epsilon < K^\sigma.$$

0.4 The Stress Form of The Piezoelectric Matrix

Given the strain form of the piezoelectric equations, we can convert to the stress form and give expressions for the three independent \mathbf{e} tensor parameters. In matrix form we have

$$\varepsilon = d^T E + c^{-1} \sigma$$

$$P = \epsilon_0 \chi E + d \sigma$$

From the first equation multiplying by the elasticity matrix c we have

$$c\varepsilon = cd^T E + \sigma,$$

so solving for the stress matrix σ we have

$$\sigma = c\varepsilon - cd^T E$$

$$= c\varepsilon - eE,$$

where the matrix e is defined as the product of c and the transpose of d

$$e = cd^T.$$

We have

$$e^T = (cd^T)^T = c^T d = cd$$

The last equality follows because c is symmetric. From the second equation

$$D = P + \epsilon_0 E = \epsilon_0 \chi E + d \sigma + \epsilon_0 E$$

$$= \epsilon_0 (\chi + I) E + d \sigma$$

$$= \epsilon^\sigma E + d \sigma,$$

where ϵ^σ is the permittivity at constant stress (unclamped permittivity). Substituting the expression for the stress σ from above

$$D = \epsilon^\sigma E + d(c\varepsilon - eE)$$

$$= (\epsilon^\sigma - de)E + dc\varepsilon$$

$$= \epsilon^\varepsilon E + e^T \varepsilon,$$

where

$$\epsilon^\varepsilon = \epsilon^\sigma - de$$

is the permittivity at constant strain (clamped permittivity). In the previous section we saw that the elastic matrix c has 5 independent parameters and d has three independent parameters. We have

$$c = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(c_{11} - c_{12}) \end{bmatrix},$$

and

$$d = \begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{bmatrix}.$$

Then the piezoelectric stress matrix is

$$e = cd^T = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(c_{11} - c_{12}) \end{bmatrix} \begin{bmatrix} 0 & 0 & d_{31} \\ 0 & 0 & d_{31} \\ 0 & 0 & d_{33} \\ 0 & d_{15} & 0 \\ d_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & c_{11}d_{31} + c_{12}d_{31} + c_{13}d_{33} \\ 0 & 0 & c_{12}d_{31} + c_{11}d_{31} + c_{13}d_{33} \\ 0 & 0 & c_{13}d_{31} + c_{13}d_{31} + c_{33}d_{33} \\ 0 & c_{44}d_{15} & 0 \\ c_{44}d_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & e_{13} \\ 0 & 0 & e_{13} \\ 0 & 0 & e_{33} \\ 0 & e_{42} & 0 \\ e_{42} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So the three independent piezo e parameters are e_{13}, e_{33}, e_{42} . (Warning! The e matrix is sometimes defined as the transpose of this e matrix.)

Explicitly we have

$$e_{13} = (c_{11} + c_{12})d_{31} + c_{13}d_{33}$$

$$e_{33} = (c_{13} + c_{13})d_{31} + c_{33}d_{33}$$

and

$$e_{42} = c_{44}d_{15}.$$

These equations are easily solved for d_{33}, d_{31} and d_{15} in terms of the three independent e and five independent c coefficients.

0.5 Three Systems of Notation For The Piezo-electric Equations: Physics (IRE), ABAQUS, and ANSYS

The index function $f(ij)$, which maps two indices to one, in the way that is conventional in elasticity theory, was defined in the previous section, as was $g(ij)$. Here is a table showing the notation most commonly used in physics (References, Nye, Auld, Cady, IRE (The Institute of Radio Engineers Standard) ANSYS Procedures p8-15,), and in the finite element systems ABAQUS (Reference: Theory manual, section 3.10.1), and ANSYS (Reference: Theory manual, Piezoelectrics chapter).

Variable	Physics	ABAQUS	ANSYS
Electric Field Vector	E_i	E_i	E_i
Electric Polarization Vector	P_i	P_i	P_i
Electric Displacement Vector	D_i	q_i	D_i
Stress Tensor	σ_{ij}	σ_{ij}	σ_{ij}
Stress Vector	$\sigma_k = \sigma_{f(ij)}$	-	T_k
Strain Tensor	ε_{ij}	ε_{ij}	-
Strain Vector	ε_k	-	S_k
Electric Permittivity Tensor	ϵ_{ij}	D_{ij}^ϕ	ϵ_{ij}
Electric Susceptibility Tensor	χ_{ij}	-	-
Elasticity Tensor	c_{ijkl}	D_{ijkl}	c_{ijkl}
Elasticity Matrix	$c_{mn} = c_{f(ij)f(kl)}$	-	c_{mn}
Inverse Elasticity Tensor	c'_{ijkl}	-	c'_{ijkl}
Piezoelectric Tensor (strain)	$d_{i,jk}$	$d_{i,jk}^\phi$	$d_{i,jk}$
Piezoelectric Matrix (strain)	$d_{i,m} = d_{i,f(jk)}$	$d_{i,m} = d_{i,g(jk)}$	$d_{i,m}$
Piezoelectric Tensor (stress)	$e_{i,jk}$	$e_{i,jk}^\phi$	$e_{jk,i}$
Piezoelectric Matrix (stress)	$e_{i,m} = e_{i,f(jk)}$	$e_{i,m} = e_{i,g(jk)}$	$e_{m,i}$

The engineering strain coefficients are written as γ_{ij} . They differ from the ε_{ij} , in that the shear coefficients are doubled in value.

Note: the physics notation for e_{ijk} is defined in equation 8.40 of Auld. Nye does not introduce the piezoelectric e coefficients. The Cady book is very old and one can obtain piezoelectric constant notation only by implication.

Note: The ANSYS definition of the e matrix and tensor is the transpose of the physics and ABAQUS definition. Thus the ANSYS e matrix is 6 by 3, whereas the physics e matrix is 3 by 6.

The two ABAQUS tensor equations are:

$$\sigma_{ij} = D_{ijkl}\varepsilon_{kl} - e_{mij}^\phi E_m$$

$$\text{(or alternately: } \sigma_{ij} = D_{ijkl}\varepsilon_{kl} - D_{ijkl}d_{mkl}^\phi E_m)$$

$$q_i = e_{ijk}^\phi + D_{ij}^\phi E_j.$$

The ANSYS matrix equations are:

$$T = cS - eE$$

$$D = e^T S + \epsilon E.$$

The Physics matrix equations are:

$$\sigma = c\varepsilon - e^T E,$$

$$D = e\varepsilon + \epsilon E.$$

Again note that the ANSYS e definition and the physics e definition are transposes of one another.

Here are the ANSYS matrices written out:

$$T = \sigma = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \dots \\ \sigma_6 \end{bmatrix}$$

$$S = \varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \dots \\ \varepsilon_6 \end{bmatrix}$$

$$E = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

$$D = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix}$$

$$c = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{21} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{31} & c_{32} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix}$$

$$e = \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ e_{21} & e_{22} & e_{23} \\ e_{31} & e_{32} & e_{33} \\ e_{41} & e_{42} & e_{43} \\ e_{51} & e_{52} & e_{53} \\ e_{61} & e_{62} & e_{63} \end{bmatrix}$$

$$\epsilon = \begin{bmatrix} \epsilon_{11} & 0 & 0 \\ 0 & \epsilon_{22} & 0 \\ 0 & 0 & \epsilon_{22} \end{bmatrix}$$

We will show in the next section that (physics notation)

$$e_{ijk} = c_{jklm} d_{ilm}.$$

0.6 Deriving the Tensor Stress Form of the Piezoelectric Equations From the Strain Form

We have already done this derivation in a previous section for the matrix forms of the piezoelectric equations. Here we will repeat this for the tensor forms of the equations. We start with the strain form of the piezoelectric equations

$$\varepsilon_{ij} = E_k d_{kij} + c'_{ijkl} \sigma_{kl},$$

and

$$P_i = \epsilon_0 \chi_{ij} E_j + d_{ijk} \sigma_{jk}.$$

Let c_{pqij} be the elasticity tensor. Because c' is its inverse, we have

$$c_{pqij} c'_{ijkl} = \delta_k^p \delta_l^q.$$

Then multiplying the first equation by c_{pqij} and summing we get

$$c_{pqij} \varepsilon_{ij} = c_{pqij} E_k d_{kij} + c_{pqij} c'_{ijkl} \sigma_{kl},$$

which becomes

$$c_{pqij}\varepsilon_{ij} = c_{pqij}E_k d_{kij} + \sigma_{pq}.$$

We let

$$e_{kpq} = c_{pqij}d_{kij}.$$

Then

$$\sigma_{pq} = c_{pqrs}\varepsilon_{rs} - e_{mpq}E_m.$$

The equation relating electric displacement D , electric polarization P , and the macro electric field E is

$$D_i = P_i + \epsilon_0 E_i.$$

Substituting for the polarization from the second piezoelectric equation above, we have

$$D_i = \epsilon_0 \chi_{ij} E_j + d_{ijk} \sigma_{jk} + \epsilon_0 E_i.$$

We write this as

$$\begin{aligned} D_i &= \epsilon_0 (\chi_{ij} + \delta_{ij}) E_j + d_{ijk} \sigma_{jk} \\ &= \epsilon_{ij}^\sigma E_j + d_{ipq} \sigma_{pq}, \end{aligned}$$

where ϵ_{ij}^σ is the zero stress (unclamped) permittivity. Substituting our previously derived expression for the stress, we get

$$\begin{aligned} D_i &= \epsilon_{im}^\sigma E_m + d_{ipq} (c_{pqrs} \varepsilon_{rs} - e_{mpq} E_m) \\ &= (\epsilon_{im}^\sigma - d_{ipq} e_{mpq}) E_m + d_{ipq} c_{pqrs} \varepsilon_{rs} \\ &= e_{irs} \varepsilon_{rs} + \epsilon_{im}^\varepsilon E_m, \end{aligned}$$

where

$$\epsilon_{im}^\varepsilon = \epsilon_{im}^\sigma - d_{ipq} e_{mpq}$$

is the zero strain (clamped) permittivity. We have assumed that $c_{pqrs} = c_{rspq}$. This is the case when the internal energy of the material is a function of the values σ , ε , D , and E , and not dependent on the path through which the state was reached, that is, when the internal energy is an exact differential in the thermodynamic sense. Therefore redefining the dummy indices, we have the stress version of the piezoelectric equations:

$$\sigma_{ij} = c_{ijkl} \varepsilon_{kl} - E_k e_{kij},$$

and

$$D_i = e_{ijk}\varepsilon_{jk} + \epsilon_{ij}^{\varepsilon}E_j.$$

The matrix forms of these equations are:

$$\sigma = c\varepsilon - e^T E,$$

$$D = e\varepsilon + \epsilon^{\varepsilon} E.$$

0.7 Orthotropic Material

For the basic equations of elasticity, see Jim Emery **Elasticity**, (See files: elastic.tex, elastic.ps).

An orthotropic problem is one in which the material properties are symmetric with respect to 3 mutually orthogonal planes (See Sokolnikoff p62). A piezoelectric ceramic such as PZT is orthotropic, but has even more symmetry. It is invariant to any rotation about the poled axis. By applying various symmetry transformations, and invariance properties, one finds that many of the elastic constants must be zero. The elastic matrix becomes

$$c = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{21} & c_{22} & c_{23} & 0 & 0 & 0 \\ c_{31} & c_{32} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix}.$$

The inverse matrix is

$$c' = c^{-1} = \begin{bmatrix} 1/E_1 & -\nu_{21}/E_2 & -\nu_{31}/E_3 & 0 & 0 & 0 \\ -\nu_{12}/E_1 & 1/E_2 & -\nu_{32}/E_3 & 0 & 0 & 0 \\ -\nu_{13}/E_1 & -\nu_{23}/E_2 & 1/E_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/(2\mu_{32}) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/(2\mu_{31}) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/(2\mu_{12}) \end{bmatrix}.$$

The Poisson ratio, ν_{ij} , is the ratio of the negative of the strain in the x_j direction, to the strain in the x_i direction, for a normal stress in the x_i direction. For example, if the only nonzero stress component is σ_1 , then

$$\begin{aligned}
& \begin{bmatrix} 1/E_1 & -\nu_{21}/E_2 & -\nu_{31}/E_3 & 0 & 0 & 0 \\ -\nu_{12}/E_1 & 1/E_2 & -\nu_{32}/E_3 & 0 & 0 & 0 \\ -\nu_{13}/E_1 & -\nu_{23}/E_2 & 1/E_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/(2\mu_{32}) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/(2\mu_{31}) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/(2\mu_{12}) \end{bmatrix} \begin{bmatrix} \sigma_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\
& = \begin{bmatrix} \sigma_1/E_1 \\ (-\sigma_1\nu_{12})/E_1 \\ (-\sigma_1\nu_{13})/E_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\
& = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ 0 \\ 0 \\ 0 \end{bmatrix}.
\end{aligned}$$

So that

$$\nu_{12} = \frac{-\varepsilon_2}{\varepsilon_1},$$

and

$$\nu_{13} = \frac{-\varepsilon_3}{\varepsilon_1},$$

By the symmetry of c , we have

$$\nu_{ij}/E_i = \nu_{ji}/E_j.$$

So there are three independent values of Poisson's ratio. We can take these to be

$$\nu_{12}, \nu_{13}, \nu_{23},$$

or

$$\nu_{21}, \nu_{31}, \nu_{32}.$$

For the case of poled piezoelectric ceramics poled in the 3 direction, the best choice is

$$\nu_{21}, \nu_{31}, \nu_{32}.$$

It is clear that in that case

$$\nu_{31} = \nu_{32},$$

so that there are only two independent poisson ratio's, which we may take as

$$\nu_{21}, \nu_{31}.$$

Also we see that ν_{31} is the poisson ratio stated for piezoceramics and is said to be about

$$.31$$

If the engineering strain vector is used then the shear strains are twice the physics shear strains, so that the 2 multiplying a shear modulus is suppressed. The engineering shear strain is equal to the shear angle, the physics shear strain is equal to one half the angle.

The notation for shear modulus μ_{ij} depends upon which of the index mappings f or g is used in assigning the stress-strain tensor indices to the vector indices.

0.8 Poling and Piezoelectric Ceramics

See:

Newnham R E, Trolier-McKinstry S, Giniewicz J R
Piezoelectric, Pyroelectric and Ferroic Crystals
J. Material Education, 15, 189-223 (1993),
(reprint: Penn State MRL annual report to ONR 1994, Appendix 1)

For the nature of the strain vs electric field curve, the nature of domains in PZT, and the morphotropic phase boundary between tetragonal PbTiO₃ and rhombohedral PbZrO₃ ferroelectric phases, see:

Subbarao E C, Srikanth V, Cao W, Cross L E
Domain Switching And Microcracking During
Poling Of Lead Zirconate Titanate Ceramics

Ferroelectrics 1993 V 145 pp. 271-281
(reprinted: Materials For
Adaptive Structural Acoustic Control, Office
Of Navel Research, Penn State Materials
Research Laboratory, L. Eric Cross ed, V1 1994, appendix 15).

The morphotropic phase boundary is the boundary between two different structures of the piezoelectric material. We note that “morph” is a root meaning form, and “trop” is a root meaning to turn or change. Hence morphotropic means: a change in form.

Consider the equation

$$P_i = d_{ij}\sigma_j.$$

Let the poling direction be the z axis. Then the domains will be randomly directed in the xy plane. Hence the ceramic will be orthotropic relative to the xy plane. Because of this symmetry, the piezoelectric matrix takes the form (see Jaffe, Cook, Jaffe **Piezoelectric Ceramics**, p20)

$$d = \begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{24} & 0 & 0 \\ d_{31} & d_{32} & d_{33} & 0 & 0 & 0 \end{bmatrix}.$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{bmatrix}.$$

The elasticity tensor takes the form

$$c = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{22} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(c_{11} - c_{12}) \end{bmatrix}.$$

The dielectric tensor is

$$\begin{bmatrix} K_1 & 0 & 0 \\ 0 & K_1 & 0 \\ 0 & 0 & K_3 \end{bmatrix}.$$

Suppose we have $d_{31} = d_{32}, d_{33}, d_{15} = d_{24}$, and $d_{ij} = 0$, otherwise. Then we have

$$\begin{aligned}d_{311} &= d_{31}, \\d_{322} &= d_{32}, \\d_{333} &= d_{33}, \\d_{113} &= d_{15}/2, \\d_{131} &= d_{15}/2, \\d_{223} &= d_{24}/2, \\d_{232} &= d_{24}/2,\end{aligned}$$

and $d_{ijk} = 0$ otherwise.

0.9 The Equality of Direct and Converse Piezo Coefficients

This equality may be shown by applying the first and second laws of thermodynamics. Let the stress coefficients σ_{ij} , the electric field components E_i and the temperature T be the state variables. Then write differential expressions for the strain de_{ij} , the electric displacement dD_i , and the entropy dS . Write the internal energy change as

$$dU = \sigma_{ij}de_{ij} + E_idD_i + TdS.$$

Introduce the function

$$\Phi = U - \sigma_{ij}e_{ij} - E_iD_i - TS.$$

Compute the differential of Φ using the last two equations, and also write it in the general partial derivative form. Then equate the partial derivatives to coefficients of the equations. For example

$$\frac{\partial\Phi}{\partial E_i} = -D_i.$$

Differentiate to equate derivatives of e_{ij} , D_i , or S to second order partial derivatives of Φ . This shows for example that the coefficients in the direct piezoelectric effect are equal to the coefficients of the converse piezoelectric effect. See Nye p179.

0.10 Typical Values of Piezoelectric Parameters

Dielectric constant

$$K = 200 = \frac{\epsilon}{\epsilon_0}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{farad/meter.}$$

K^T dielectric constant at constant stress. K^S dielectric constant at zero strain (constrained piezoelectric coefficient)

$$d_{33} = 200 \times 10^{-12} \text{coulomb/newton (or meter/volt).}$$

$$d_{31} = -100 \times 10^{-12} \text{coulomb/newton (or meter/volt).}$$

$$d_{15} = 400 \times 10^{-12} \text{coulomb/newton (or meter/volt).}$$

g_{ij} voltage coefficients.

$$k_{ij} = \epsilon g_{ij}$$

Poisson's ratio is approximately .31 for all ceramics. Curie temperature

$$T_c = 300.$$

f_r = resonant frequency. Typical value is 70 kHz.

Panasonic motor.

Operating voltage 22 volts.

Rated output 1 watt.

Torque 250 gf.cm (gf gram force)

250 gf cm (pound/454 gf)(inch/2.54 cm)(16 ounce/pound) = 3.47 inch.ounce

Current .4 amp at 5 to 10 volts

Input power 2 to 4 watts

Voltage of control circuit 12 volts

250 g.cm (kg/1000 g)(9.8 m/s.s)(m/100 cm) = .0245 N.m

Angular velocity = $4002\pi/(min)(min/60sec) = 10.47(1/s)$

Power = (.0245)(10.47) = .2565 watt

Speed 400 rpm, no-load 600 rpm.

f_a = antiresonant frequency (parallel resonance)

Electromagnetic coupling constant

$k = (\text{electrical energy stored})/(\text{mechanical energy stored}).$

$k_p = .2$ to $.8 =$ planar coupling factor of a thin disk.

$Q_m =$ quality factor.

Example: Thickness

$$l = .030\text{inches} = 7.63 \times 10^{-4}m$$

$$v = 400\text{volts.}$$

$$E = v/l = 3.67 \times 10^6 v/m.$$

$$e = dE = (200 \times 10^{-12})(3.67 \times 10^6) = 7.34 \times 10^{-4}.$$

$$\Delta z = el = 22\mu\text{inches}$$

0.11 Piezoelectric Current and Impedance From A Finite Element Calculation

Let D be the electric displacement and ρ the charge density. We can use one of Maxwell's equations

$$\nabla \cdot D = \rho,$$

to show that the surface charge density on a conductor equals the normal component D_n of D outside of the conductor. We enclose a conducting surface with a thin pillbox of area A , volume V , and thickness Δt . The charge contained in the volume is $A\tau$ where τ is the surface charge density. Then letting Δt go to zero, we have

$$A\tau = \int_V \rho dV = \int_V \nabla \cdot D dV = \int_{\partial V} D \cdot n ds \rightarrow D \cdot n A.$$

Then the charge density is

$$\tau = D \cdot n = D_n.$$

We compute the nodal strains ε_{ij} from the nodal displacements u_i at a face of an element.

From the finite element nodal potentials ϕ_n , we compute derivatives of the shape functions, and then the gradient of the potential. Then we obtain the electric field E_i as the negative gradient of the potential,

$$E = -\nabla \phi.$$

We compute the electric displacement from the piezoelectric equation

$$D_i = e_{ijk}\varepsilon_{jk} + K_{ij}E_j.$$

Then we compute the normal component D_n . This gives the surface charge density. Multiplying by the area A , we get the charge q . We sum up the charges on the boundaries of the elements to get the total electrode charge. If we calculate the charge as a function of time, we can fit it to a function

$$Q_0 \cos(\omega t + \psi),$$

and thus to the real part of the exponential

$$Q_0 \exp(j(\omega t + \psi)).$$

Differentiating with respect to time we get the current

$$I = I_0 \exp(j(\omega t + \psi)).$$

Then if V is the applied voltage, the impedance is

$$Z = V/I.$$

We can also compute energy storage and dissipation in the finite element model, and relate it to energy storage and dissipation in an equivalent circuit. We can obtain or compute the energy density of the electric field

$$\frac{D \cdot E}{2},$$

the elastic strain energy, the kinetic energy, heat dissipation, and frictional contact energy. The kinetic energy will correspond to inductor energy in the equivalent circuit, the strain energy and the electric field energy will correspond to capacitive energy, and the dissipation energy will correspond to a resistance loss. If the energies correspond, then the equivalent circuit is valid.

0.12 Piezoelectric Cube Example

Consider a piezoelectric cube of side a . Let thin metal plates be located at $z = 0$ and $z = a$. Let there be a voltage V on these plates. Let there be a horizontal compressive forces F in the y direction acting on the $y = 0$, and the $y = a$ faces of the block. The electric field is

$$E = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ V/a \end{bmatrix}$$

The stress is

$$\sigma = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} = \begin{bmatrix} F/a^2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

An average dielectric constant for PZT is

$$K = 200 = \frac{\epsilon}{\epsilon_o} = 1 + \chi.$$

We have

$$\epsilon_o = 8.85 \times 10^{10} (C^2/NM^2).$$

Then the permittivity is

$$\epsilon = 1.77 \times 10^{-9}.$$

If we take the material to be isotropic, the polarization equation becomes

$$P = \epsilon_o \chi E + \begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & 0 & d_{25} & 0 \\ d_{31} & d_{32} & d_{33} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix}$$

We have

$$d_{31} = d_{32} = -100 \times 10^{-12} C/N \text{ or } M/V$$

$$d_{33} = 2100 \times 10^{-12}$$

$$d_{51} = d_{15} = 400 \times 10^{-12}$$

Because only σ_1 is nonzero, we have

$$P = \epsilon_0 \chi E + \begin{bmatrix} 0 \\ 0 \\ d_{31} \sigma_1 \end{bmatrix}.$$

Then

$$P_3 = \epsilon_0 \chi E_3 + d_{31} \sigma_1$$

$$D = \epsilon_0 E + P.$$

$$D_3 = \epsilon_0(1 + \chi)E_3 + d_{31} \sigma_1 = \epsilon_0 K E_3 + d_{31} \sigma_1.$$

The surface charge on the $z = a$ face is

$$Q = D_3 a^2.$$

The strain is

$$e = \begin{bmatrix} 0 & 0 & d_{31} \\ 0 & 0 & d_{32} \\ 0 & 0 & d_{33} \\ 0 & d_{25} & 0 \\ d_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} + \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{21} & c_{22} & c_{33} & 0 & 0 & 0 \\ c_{31} & c_{32} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix}$$

So the nonzero elements of the strain are

$$e_1 = d_{31} E_3 + \sigma_1 c_{11} = d_{31} E_3 + \frac{\sigma_1}{\xi},$$

$$e_2 = d_{32} E_3 + \sigma_1 c_{21} = d_{32} E_3 - \frac{\sigma_1 \nu}{\xi},$$

$$e_3 = d_{33} E_3 + \sigma_1 c_{31} = d_{33} E_3 - \frac{\sigma_1 \nu}{\xi},$$

where ξ is Young's modulus, and ν is Poisson's ratio.

0.13 Generating Ferroelectric Hysteresis Curves

Suppose we arrange two capacitors in series, one C_1 containing a ferroelectric dielectric, and the second C_2 a nonferroelectric dielectric. Let us connect in series an alternating high voltage source. Let V_1 be the voltage across capacitor C_1 . This is a measure of the E field in the ferroelectric capacitor C_1 . The charge on the plates of C_1 is proportional to the D field (actually the D field is equal to the surface charge density). But the charge on C_1 is equal to the charge on C_2 and moves from C_1 to C_2 and back again during a cycle of the applied alternating voltage. But the second capacitor is nonferroelectric and its dielectric is linear, so that the voltage V_2 across the second capacitor is proportional to charge and hence to the D field of the ferroelectric capacitor. Therefore, if we connect V_1 across the horizontal plates of an oscilloscope, and V_2 across the vertical plates, we will see the hysteresis curve. The polarization field will be approximately equal to D , since the permittivity ϵ for the ferroelectric is much larger than the permittivity of free space ϵ_0 .

(I think this is close to the method given by Sawyer and Tower, see bibliography.)

0.14 The Strain Hysteresis Curve

The strain vs electric field curve for a piezoelectric ceramic is a butterfly curve with the strain always of one sign, and the curve lying in the positive upper half plane.

0.15 Relaxors

A relaxor ferroelectric has some of the properties of a normal ferroelectric, but is not so permanently polarized and has a very narrow hysteresis loop. When the field is removed, the material almost returns to an unpolarized state. It requires a bias field.

See:

Cross L Eric
Relaxor Ferroelectrics: An Overview
Ferroelectrics, 1994 v151 pp. 305-320

(reprinted: annual report 1994 from MRL at Penn State to ONR v1 p305)

0.16 Electrostriction

Electrostriction is an internal stress caused by the force of the electric field on charges. It occurs in all materials, not just piezoelectric ones. A computation shows it to be related to the gradient of the dielectric constant, and in practice the stress is proportional to the square of the electric field intensity. It is very hard to measure, since the force can be masked by the force due to the free charges on the capacitor plates.

Notes and references to electrostriction: Wert, Thomson **Physics of Solids**, p. 321. Julius Adams Stratton, **Electromagnetic Theory**, pp149-151. Panofsky and Philips **Classical Electricity and Magnetism** p.95, As a term of an equation. Maxwell stress tensor. Classical derivation. The treatment in Panofsky and Phillips is based on the treatment in Becker: (formerly called Becker and Abraham) **Electromagnetic Fields and Interactions** p. 134. In Jaffe: **Piezoelectric Ceramics**, Electrostriction is mentioned on pages 15 and 78. He relates electrostriction to domain reversal. Jaffe says that the effect is significant only for ferroelectrics above the Curie point. Cady: **Piezoelectricity** defines electrostriction to be that effect that is proportional to $\|E\|^2$. See pages 4, 198, 199, 614. Cady states that electrostriction is only significant for fields above 20,000 volts per cm. See also Auld, **Acoustic Fields and Waves in Solids**, volumes I and II, who presents a nice one dimensional model of the piezoelectric effect on p265.

0.17 The Conversion Program: pzansys.ftn

This program does the conversion calculations derived in the previous sections. It requires as input the d parameters, and either the elasticity matrix, or its inverse. It computes the e parameters, and computes the clamped permittivity (strain constant) from the unclamped permittivity. It generates material property cards for the ANSYS program. We list the program in a later section. The next sections illustrate the operation of the program.

0.18 Example: Parameter Conversion Using Compliance Components

```
yonada:/users/u51195 $ pzansys
Reference: Jim Emery "Piezoelectricity",
Document location:/u51195/ps/piezoelc.ps,piezoelc.tex
Anonymous FTP: ftp.os.kcp.com /pub/emery/
Units are metric: Newton, Meter, Kilogram
stress: Newton/M^2 (Pascals)
Young's Modulus: Pascals
Elasticity coefficients: Pascals
Enter piezoelectric d constants (coulomb/newton)
The IRE convention for piezoelectric labeling is used
Type <return> for default value
Enter d33 [ 2.0000000000000001E-10]

Enter d31 [ -1.0000000000000000E-10]

Enter d15 [ 4.0000000000000001E-10]

d matrix=
0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.4000E-09 0.0000E+00
0.0000E+00 0.0000E+00 0.0000E+00 0.4000E-09 0.0000E+00 0.0000E+00
-0.1000E-09 -0.1000E-09 0.2000E-09 0.0000E+00 0.0000E+00 0.0000E+00
Do you want to see the nonzero components
of the 27 d tensor components? [n]

Define the elasticity matrix or compliance matrix:

(1)Use elastic constants to define compliance
for orthotropic material, using Young's moduli,
Poissons ratios, and Shear Moduli.

(2)Use the five components of the elastic matrix
c11,c33,c44 (c44 is ANSYS c66),c12,c13,
for a z-poled ceramic.
(Note: c22=c11, c55=c44, c66=2(c11-c12),c23=c13,
and c_{23}=c_{13}. Coefficients which are
not in the upper 3-dimensional submatrix,
not on the main diagonal, are zero.

(3)Use the five components of the compliance matrix
s11,s33,s44 (s44 is ANSYS s66),s12,s13,
for a z-poled ceramic.
(Note: s22=s11, s55=s44, s66=2(s11-s12),s23=s13,
and s_{23}=s_{13}. Coefficients which are
not in the upper 3-dimensional submatrix,
not on the main diagonal, are zero.
type <return> for default selection: [1]

Enter the orthotropic elastic constants
for a coordinate system in the principle directions
Enter Young's Modulus, in x [ 0.12800E+12]
```

Enter Young's Modulus, in y [0.12800E+12]

Enter Young's Modulus, in z [0.11000E+12]

Enter ANSYS Poisson ratios (Theory sec. 2.1)

Enter Major Poisson ratio prxy[0.30000]

Enter Major Poisson ratio prxz[0.41976]

Enter Major Poisson ratio pryz[0.41976]

Enter shear modulus g23 [0.52000E+11]

Enter shear modulus g13 [0.52000E+11]

Enter shear modulus g12 [0.12200E+12]

Compliance matrix (elasticity inverse) cinv=
0.7813E-11 -0.2344E-11 -0.3279E-11 0.0000E+00 0.0000E+00 0.0000E+00
-0.2344E-11 0.7813E-11 -0.3279E-11 0.0000E+00 0.0000E+00 0.0000E+00
-0.3279E-11 -0.3279E-11 0.9091E-11 0.0000E+00 0.0000E+00 0.0000E+00
0.0000E+00 0.0000E+00 0.0000E+00 0.1923E-10 0.0000E+00 0.0000E+00
0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.1923E-10 0.0000E+00
0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.8197E-11

Note: In ANSYS components 4,5,6 are permuted
(A different mapping from tensor to vector is used):

ANSYS 44 is 66

ANSYS 55 is 44

ANSYS 66 is 55

The five independent compliance coefficients are:

s11= 0.78125000E-11

s33= 0.90909091E-11

s44= 0.19230769E-10

s12=-0.23437500E-11

s13=-0.32793388E-11

Computing inverse matrix

Elasticity matrix =

0.2104E+12 0.1119E+12 0.1163E+12 0.0000E+00 0.0000E+00 0.0000E+00
0.1119E+12 0.2104E+12 0.1163E+12 0.0000E+00 0.0000E+00 0.0000E+00
0.1163E+12 0.1163E+12 0.1939E+12 0.0000E+00 0.0000E+00 0.0000E+00
0.0000E+00 0.0000E+00 0.0000E+00 0.5200E+11 0.0000E+00 0.0000E+00
0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.5200E+11 0.0000E+00
0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.1220E+12

Orthotropic constants from compliance matrix:

Young's Modulus 1= 0.12800000E+12

Young's Modulus 2= 0.12800000E+12

Young's Modulus 3= 0.11000000E+12

Major Poisson ratio prxy= 0.30000000

Major Poisson ratio prxz= 0.41975537

Major Poisson ratio pryz= 0.41975537

Shear Modulus g23= 0.52000000E+11

Shear Modulus g13= 0.52000000E+11

Shear Modulus g12= 0.12200000E+12

Do you want to see the nonzero components
of the 81 elasticity tensor components of c? [n]

piezoelectric (stress) tensor coefficients
Do you want to see the nonzero components
of the 27 piezoelectric tensor components of e? [n]

Piezoelectric e matrix=
0.0000E+00 0.0000E+00 -8.977
0.0000E+00 0.0000E+00 -8.977
0.0000E+00 0.0000E+00 15.52
0.0000E+00 20.80 0.0000E+00
20.80 0.0000E+00 0.0000E+00
0.0000E+00 0.0000E+00 0.0000E+00

Second calculation:

e13= -8.977066554171117
e33= 15.52345455897376
e42= 20.800000000000000

Note: ANSYS e rows are permuted

ANSYS row 4 is row 6

ANSYS row 5 is row 4

ANSYS row 6 is row 5

ANSYS Piezoelectric e matrix=
0.0000E+00 0.0000E+00 -8.977
0.0000E+00 0.0000E+00 -8.977
0.0000E+00 0.0000E+00 15.52
0.0000E+00 0.0000E+00 0.0000E+00
0.0000E+00 20.80 0.0000E+00
20.80 0.0000E+00 0.0000E+00

d times e =

0.8320E-08 0.0000E+00 0.0000E+00
0.0000E+00 0.8320E-08 0.0000E+00
0.0000E+00 0.0000E+00 0.4900E-08

Enter the unclamped dielectric constant K11

[1300.0000]

Enter the unclamped dielectric constant K33

[1000.0000]

Unclamped electric permittivity tensor=

0.1151E-07 0.0000E+00 0.0000E+00
0.0000E+00 0.1151E-07 0.0000E+00
0.0000E+00 0.0000E+00 0.8850E-08

Clamped electric permittivity tensor=

0.3185E-08 0.0000E+00 0.0000E+00
0.0000E+00 0.3185E-08 0.0000E+00
0.0000E+00 0.0000E+00 0.3950E-08

Enter density Kg/m³ [7500]

/COM

/COM Material properties

/COM Young's Moduli

EX, 2, 0.12800000E+12

EY, 2, 0.12800000E+12

EZ, 2, 0.11000000E+12

```

/COM ANSYS "Major" Poisson ratios
PRXY, 2, 0.30000000
PRXZ, 2, 0.41975537
PRYZ, 2, 0.41975537
/COM Density Kg/m^3
/COM DENS, 2, 0.75000000E-02
/COM clamped Permittivities
MP,PERX, 2, 0.31850000E-08
MP,PERY, 2, 0.31850000E-08
MP,PERZ, 2, 0.39498958E-08
/COM ANSYS Piezoelectric "e" matrix
TB,PIEZ,3
TBDATA, 3, -8.9770666
TBDATA, 6, -8.9770666
TBDATA, 9, 15.523455
TBDATA, 14, 20.800000
TBDATA, 16, 20.800000

```

0.19 Example: Parameter Conversion Using Four Independent Elasticity Components

```

yonada:/users/u51195 $ pzansys
Reference: Jim Emery "Piezoelectricity", (piezoelc.ps)
Document location:/u51195/ps/piezoelc.ps,piezoelc.tex
Anonymous FTP: ftp.os.kcp.com /pub/emery/
Units are metric: Newton, Meter, Kilogram
stress: Newton/M^2 (Pascals)
Young's Modulus: Pascals
Elasticity coefficients: Pascals, and so on)
Enter piezoelectric d constants (coulomb/newton)
The IRE convention for piezoelectric labeling is used
Type <return> for default value
Enter d33 [ 2.0000000000000001E-10]

Enter d31 [ -1.0000000000000000E-10]

Enter d15 [ 4.0000000000000001E-10]

d matrix=
0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.4000E-09 0.0000E+00
0.0000E+00 0.0000E+00 0.0000E+00 0.4000E-09 0.0000E+00 0.0000E+00
-0.1000E-09 -0.1000E-09 0.2000E-09 0.0000E+00 0.0000E+00 0.0000E+00
Do you want to see the nonzero components
of the 27 d tensor components? [n]

Define the elasticity matrix or compliance matrix:

(1)Use elastic constants to define compliance
for orthotropic material, using Young's moduli,
Poissons ratios, and Shear Moduli.

(2)Use the five components of the elastic matrix

```

c11,c33,c44 (c44 is ANSYS c66),c12,c13,
 for a z-poled ceramic.
 (Note: c22=c11, c55=c44, c66=2(c11-c12),c23=c13,
 and c_{23}=c_{13}. Coefficients which are
 not in the upper 3-dimensional submatrix,
 not on the main diagonal, are zero.

(3)Use the five components of the compliance matrix
 s11,s33,s44 (s44 is ANSYS s66),s12,s13,
 for a z-poled ceramic.
 (Note: s22=s11, s55=s44, s66=2(s11-s12),s23=s13,
 and s_{23}=s_{13}. Coefficients which are
 not in the upper 3-dimensional submatrix,
 not on the main diagonal, are zero.
 type <return> for default selection: [1]

2

Enter c11 [0.13200E+12]

Enter c33 [0.11500E+12]

Enter c44 (ANSYS c66) [0.52000E+11]

Enter c12 [0.71000E+11]

Enter c13 [0.73000E+11]

Elasticity matrix c=
 0.1320E+12 0.7100E+11 0.7300E+11 0.0000E+00 0.0000E+00 0.0000E+00
 0.7100E+11 0.1320E+12 0.7300E+11 0.0000E+00 0.0000E+00 0.0000E+00
 0.7300E+11 0.7300E+11 0.1150E+12 0.0000E+00 0.0000E+00 0.0000E+00
 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.5200E+11 0.0000E+00
 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.5200E+11 0.0000E+00
 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.1220E+12

Note: In ANSYS components 4,5,6 are permuted

ANSYS 44 is 66

ANSYS 55 is 44

ANSYS 66 is 55

Compliance matrix=

0.1273E-10 -0.3665E-11 -0.5754E-11 0.0000E+00 0.0000E+00 0.0000E+00
 -0.3665E-11 0.1273E-10 -0.5754E-11 0.0000E+00 0.0000E+00 0.0000E+00
 -0.5754E-11 -0.5754E-11 0.1600E-10 0.0000E+00 0.0000E+00 0.0000E+00
 0.0000E+00 0.0000E+00 0.0000E+00 0.1923E-10 0.0000E+00 0.0000E+00
 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.1923E-10 0.0000E+00
 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.8197E-11

Orthotropic constants from compliance matrix:

Young's Modulus 1= 0.78561263E+11

Young's Modulus 2= 0.78561263E+11

Young's Modulus 3= 0.62497537E+11

Major Poisson ratio prxy= 0.28788955

Major Poisson ratio prxz= 0.45203533

Major Poisson ratio pryz= 0.45203533

Shear Modulus g23= 0.52000000E+11

Shear Modulus g13= 0.52000000E+11

Shear Modulus g12= 0.12200000E+12

Do you want to see the nonzero components
of the 81 elasticity tensor components of c? [n]

piezoelectric (stress) tensor coefficients
Do you want to see the nonzero components
of the 27 piezoelectric tensor components of e? [n]

Piezoelectric e matrix=
0.0000E+00 0.0000E+00 -5.700
0.0000E+00 0.0000E+00 -5.700
0.0000E+00 0.0000E+00 8.400
0.0000E+00 20.80 0.0000E+00
20.80 0.0000E+00 0.0000E+00
0.0000E+00 0.0000E+00 0.0000E+00

Second calculation:

e13= -5.700000000000001
e33= 8.400000000000000
e42= 20.800000000000000

Note: ANSYS e rows are permuted

ANSYS row 4 is row 6

ANSYS row 5 is row 4

ANSYS row 6 is row 5

ANSYS Piezoelectric e matrix=
0.0000E+00 0.0000E+00 -5.700
0.0000E+00 0.0000E+00 -5.700
0.0000E+00 0.0000E+00 8.400
0.0000E+00 0.0000E+00 0.0000E+00
0.0000E+00 20.80 0.0000E+00
20.80 0.0000E+00 0.0000E+00

d times e =

0.8320E-08 0.0000E+00 0.0000E+00
0.0000E+00 0.8320E-08 0.0000E+00
0.0000E+00 0.0000E+00 0.2820E-08

Enter the unclamped dielectric constant K11
[1300.0000]

Enter the unclamped dielectric constant K33
[1000.0000]

Unclamped electric permittivity tensor=

0.1151E-07 0.0000E+00 0.0000E+00
0.0000E+00 0.1151E-07 0.0000E+00
0.0000E+00 0.0000E+00 0.8850E-08

Clamped electric permittivity tensor=

0.3185E-08 0.0000E+00 0.0000E+00
0.0000E+00 0.3185E-08 0.0000E+00
0.0000E+00 0.0000E+00 0.6030E-08

Enter density Kg/m³ [7500]

/COM

/COM Material properties

/COM Young's Moduli

EX, 2, 0.78561263E+11

EY, 2, 0.78561263E+11

EZ, 2, 0.62497537E+11

```

/COM ANSYS "Major" Poisson ratios
PRXY, 2, 0.28788955
PRXZ, 2, 0.45203533
PRYZ, 2, 0.45203533
/COM Density Kg/m^3
/COM DENS, 2, 0.75000000E-02
/COM clamped Permittivities
MP,PERX, 2, 0.31850000E-08
MP,PERY, 2, 0.31850000E-08
MP,PERZ, 2, 0.60300000E-08
/COM ANSYS Piezoelectric "e" matrix
TB,PIEZ,3
TBDATA, 3, -5.7000000
TBDATA, 6, -5.7000000
TBDATA, 9, 8.4000000
TBDATA, 14, 20.800000
TBDATA, 16, 20.800000

```

0.20 Example, Orthotropic Elastic Constants

```

Reference: Jim Emery "Piezoelectricity", (piezoelc.ps)
Document location:/u51195/ps/piezoelc.ps,piezoelc.tex
Anonymous FTP: ftp.os.kcp.com /pub/emery/
Units are metric: Newton, Meter, Kilogram
stress: Newton/M^2 (Pascals)
Young's Modulus: Pascals
Elasticity coefficients: Pascals, and so on)
Enter piezoelectric d constants (coulomb/newton)
The IRE convention for piezoelectric labeling is used
Type <return> for default value
Enter d33 [ 2.0000000000000001E-10]
Enter d31 [ -1.0000000000000000E-10]
Enter d15 [ 4.0000000000000001E-10]
d matrix=
0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.4000E-09 0.0000E+00
0.0000E+00 0.0000E+00 0.0000E+00 0.4000E-09 0.0000E+00 0.0000E+00
-0.1000E-09 -0.1000E-09 0.2000E-09 0.0000E+00 0.0000E+00 0.0000E+00
Do you want to see the nonzero components
of the 27 d tensor components? [n]
Define the elasticity matrix or compliance matrix:

(1)Use elastic constants to define compliance
for orthotropic material, using Young's moduli,
Poissons ratios, and Shear Moduli.

(2)Use the five components of the elastic matrix
c11,c33,c44 (c44 is ANSYS c66),c12,c13,
for a z-poled ceramic.
(Note: c22=c11, c55=c44, c66=2(c11-c12),c23=c13,
and c_{23}=c_{13}. Coefficients which are
not in the upper 3-dimensional submatrix,
not on the main diagonal, are zero.

```

```

(3)Use the five components of the compliance matrix
s11,s33,s44 (s44 is ANSYS s66),s12,s13,
for a z-poled ceramic.
(Note: s22=s11, s55=s44, s66=2(s11-s12),s23=s13,
and s_{23}=s_{13}. Coefficients which are
not in the upper 3-dimensional submatrix,
not on the main diagonal, are zero.
type <return> for default selection: [1]
Enter the orthotropic elastic constants
for a coordinate system in the principle directions
Enter Young's Modulus, in x [ 0.12800E+12]
Enter Young's Modulus, in y [ 0.12800E+12]
Enter Young's Modulus, in z [ 0.11000E+12]

Enter ANSYS Poisson ratios (Theory sec. 2.1)
Enter Major Poisson ratio prxy[ 0.30000 ]
Enter Major Poisson ratio prxz[ 0.41976 ]
Enter Major Poisson ratio pryz[ 0.41976 ]
Enter shear modulus g23 [ 0.52000E+11]
Enter shear modulus g13 [ 0.52000E+11]
Enter shear modulus g12 [ 0.12200E+12]
Compliance matrix (elasticity inverse) cinv=
 0.7813E-11 -0.2344E-11 -0.3279E-11 0.0000E+00 0.0000E+00 0.0000E+00
-0.2344E-11 0.7813E-11 -0.3279E-11 0.0000E+00 0.0000E+00 0.0000E+00
-0.3279E-11 -0.3279E-11 0.9091E-11 0.0000E+00 0.0000E+00 0.0000E+00
0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.1923E-10 0.0000E+00
0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.1923E-10 0.0000E+00
0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.8197E-11
Note: In ANSYS components 4,5,6 are permuted
(A different mapping from tensor to vector is used):
ANSYS 44 is 66
ANSYS 55 is 44
ANSYS 66 is 55
The five independent compliance coefficients are:
s11= 0.78125000E-11
s33= 0.90909091E-11
s44= 0.19230769E-10
s12=-0.23437500E-11
s13=-0.32793388E-11
Computing inverse matrix
Elasticity matrix =
 0.2104E+12 0.1119E+12 0.1163E+12 0.0000E+00 0.0000E+00 0.0000E+00
 0.1119E+12 0.2104E+12 0.1163E+12 0.0000E+00 0.0000E+00 0.0000E+00
 0.1163E+12 0.1163E+12 0.1939E+12 0.0000E+00 0.0000E+00 0.0000E+00
 0.0000E+00 0.0000E+00 0.0000E+00 0.5200E+11 0.0000E+00 0.0000E+00
 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.5200E+11 0.0000E+00
 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.0000E+00 0.1220E+12
Orthotropic constants from compliance matrix:
Young's Modulus 1= 0.12800000E+12
Young's Modulus 2= 0.12800000E+12
Young's Modulus 3= 0.11000000E+12
Major Poisson ratio prxy= 0.30000000
Major Poisson ratio prxz= 0.41975537
Major Poisson ratio pryz= 0.41975537
Shear Modulus g23= 0.52000000E+11

```

Shear Modulus g13= 0.52000000E+11
Shear Modulus g12= 0.12200000E+12

Do you want to see the nonzero components
of the 81 elasticity tensor components of c? [n]
piezoelectric (stress) tensor coefficients
Do you want to see the nonzero components
of the 27 piezoelectric tensor components of e? [n]

Piezoelectric e matrix=
0.0000E+00 0.0000E+00 -8.977
0.0000E+00 0.0000E+00 -8.977
0.0000E+00 0.0000E+00 15.52
0.0000E+00 20.80 0.0000E+00
20.80 0.0000E+00 0.0000E+00
0.0000E+00 0.0000E+00 0.0000E+00

Second calculation:
e13= -8.977066554171117
e33= 15.52345455897376
e42= 20.800000000000000

Note: ANSYS e rows are permuted
ANSYS row 4 is row 6
ANSYS row 5 is row 4
ANSYS row 6 is row 5

ANSYS Piezoelectric e matrix=
0.0000E+00 0.0000E+00 -8.977
0.0000E+00 0.0000E+00 -8.977
0.0000E+00 0.0000E+00 15.52
0.0000E+00 0.0000E+00 0.0000E+00
0.0000E+00 20.80 0.0000E+00
20.80 0.0000E+00 0.0000E+00

d times e =
0.8320E-08 0.0000E+00 0.0000E+00
0.0000E+00 0.8320E-08 0.0000E+00
0.0000E+00 0.0000E+00 0.4900E-08

Enter the unclamped dielectric constant K11
[1300.0000]

Enter the unclamped dielectric constant K33
[1000.0000]

Unclamped electric permittivity tensor=
0.1151E-07 0.0000E+00 0.0000E+00
0.0000E+00 0.1151E-07 0.0000E+00
0.0000E+00 0.0000E+00 0.8850E-08

Clamped electric permittivity tensor=
0.3185E-08 0.0000E+00 0.0000E+00
0.0000E+00 0.3185E-08 0.0000E+00
0.0000E+00 0.0000E+00 0.3950E-08

Enter density Kg/m³ [7500]

/COM

/COM Material properties

/COM Young's Moduli

EX, 2, 0.12800000E+12

EY, 2, 0.12800000E+12

EZ, 2, 0.11000000E+12

/COM ANSYS "Major" Poisson ratios

PRXY, 2, 0.30000000

```

PRXZ, 2, 0.41975537
PRYZ, 2, 0.41975537
/COM Density Kg/m^3
/COM DENS, 2, 0.75000000E-02
/COM clamped Permittivities
MP,PERX, 2, 0.31850000E-08
MP,PERY, 2, 0.31850000E-08
MP,PERZ, 2, 0.39498958E-08
/COM ANSYS Piezoelectric "e" matrix
TB,PIEZ,3
TBDATA, 3, -8.9770666
TBDATA, 6, -8.9770666
TBDATA, 9, 15.523455
TBDATA, 14, 20.800000
TBDATA, 16, 20.800000

```

0.21 Program Listing of pzansys.ftn Program

Here is a listing of the program. The program works through tensor summations, matrix calculations, and matrix inversions. Each subroutine has a one line description and a description of the parameters.

```

% cat pzansys.ftn
c pzansys.ftn stress and strain versions of the piezoelectric tensor
c Version 2/6/97
c Author: Jim Emery (AlliedSignal Kansas City)
c for the ansys finite element program.
column 111111111222222222223333333333344444444445555555555666666666677777777778
c2345678901234567890123456789012345678901234567890123456789012345678901234567890
c Reference: Jim Emery "Piezoelectricity" (File: piezoelc.tex)
    implicit real*8 (a-h,o-z)
    dimension d(3,3,3)
    dimension e(3,3,3)
    dimension c(3,3,3,3)
    dimension dc(3,3)
    dimension cmat(6,6)
    dimension cmatinv(6,6)
    dimension a(6,6),b(6,6),cc(6,6)
    dimension dmat(3,6)
    dimension emat(6,3)
    dimension epsilon(3,3)
    dimension clamped(3,3)

    dimension ain(10)
    dimension pr(3)
    dimension ym(3)
    real*8 nu12,nu23,nu13,nu21,nu32,nu31
    character*1 cm
    character*1 reply
    character*53 cs1
    zero=0.
c piezoelectric constants (coulomb/newton)

```

```

do i=1,3
  do j=1,6
    dmat(i,j)=0.
  enddo
enddo
dmat(3,3)=200e-12
dmat(3,1)=-100e-12
dmat(1,5)=400e-12
write(*,*)'Reference: Jim Emery "Piezoelectricity", (piezoelc.ps)'
write(*,*)'Document location:/u51195/ps/piezoelc.ps,piezoelc.tex '
write(*,*)'Anonymous FTP: ftp.os.kcp.com /pub/emery/ '
write(*,*)'Units are metric: Newton, Meter, Kilogram'
write(*,*)'stress: Newton/M^2 (Pascals)'
write(*,*)'Young's Modulus: Pascals'
write(*,*)'Elasticity coefficients: Pascals, and so on)'
write(*,*)'Enter piezoelectric d constants (coulomb/newton)'
write(*,*)'The IRE convention for piezoelectric labeling is used'
write(*,*)'Type <return> for default value'
write(*,*)'Enter d33 [',dmat(3,3),']'
call readr(0,ain,nr)
if(nr .ge. 1)then
  dmat(3,3)=ain(1)
endif
write(*,*)'Enter d31 [',dmat(3,1),']'
call readr(0,ain,nr)
if(nr .ge. 1)then
  dmat(3,1)=ain(1)
endif
dmat(3,2)=dmat(3,1)
write(*,*)'Enter d15 [',dmat(1,5),']'
call readr(0,ain,nr)
if(nr .ge. 1)then
  dmat(1,5)=ain(1)
endif
dmat(2,4)=dmat(1,5)
write(*,*)' d matrix= '
do i=1,3
  write(*,'(6(g11.4,1x))')(dmat(i,j),j=1,6)
enddo
c  construct the components of the d tensor
do i=1,3
  do j=1,3
    do k=1,3
      call t2vec(j,k,jk)
      if(j .eq. k)then
        d(i,j,k)=dmat(i,jk)
      else
        d(i,j,k)=dmat(i,jk)/2.
      endif
    enddo
  enddo
enddo
write(*,*)'Do you want to see the nonzero components'
write(*,*)'of the 27 d tensor components? [n]'
read(*,'(a)')reply

```

```

if(lenstr(reply) .eq. 0)then
  reply='n'
endif
if(reply .eq. 'y')then
  do i=1,3
    do j=1,3
      do k=1,3
        if(d(i,j,k) .ne. zero)then
          write(*,'(a,i1,i1,i1,a,g15.8)')'d(' ,i,j,k,')=' ,d(i,j,k)
        endif
      enddo
    enddo
  enddo
endif

c orthotropic elastic constants
write(*,*)'Define the elasticity matrix or compliance matrix:'
write(*,*)
write(*,*)'(1)Use elastic constants to define compliance'
write(*,*)'for orthotropic material, using Young's moduli,'
write(*,*)'Poissons ratios, and Shear Moduli.'
write(*,*)
write(*,*)'(2)Use the five components of the elastic matrix'
write(*,*)'c11,c33,c44 (c44 is ANSYS c66),c12,c13, '
write(*,*)'for a z-poled ceramic.'
write(*,*)'(Note: c22=c11, c55=c44, c66=2(c11-c12),c23=c13,'
write(*,*)'and c_{23}=c_{13}. Coefficients which are'
write(*,*)'not in the upper 3-dimensional submatrix,'
write(*,*)'not on the main diagonal, are zero.'
write(*,*)
write(*,*)'(3)Use the five components of the compliance matrix'
write(*,*)'s11,s33,s44 (s44 is ANSYS s66),s12,s13,'
write(*,*)'for a z-poled ceramic.'
write(*,*)'(Note: s22=s11, s55=s44, s66=2(s11-s12),s23=s13,'
write(*,*)'and s_{23}=s_{13}. Coefficients which are'
write(*,*)'not in the upper 3-dimensional submatrix,'
write(*,*)'not on the main diagonal, are zero.'
write(*,*)'type <return> for default selection: [1]'

read(*,'(a)')reply
if(lenstr(reply) .eq. 0)then
  reply='1'
endif

c234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890

c
begin case1
if(reply .eq. '1')then
write(*,*)'Enter the orthotropic elastic constants'
write(*,*)'for a coordinate system in the principle directions'
e1=12.8e10
write(*,'(a,g12.5,a)')'Enter Young's Modulus, in x [' ,e1,']'
call readr(0,ain,nr)
if(nr .eq. 1)then

```

```

    e1=ain(1)
endif

e2=12.8e10
write(*,'(a,g12.5,a)')'Enter Young's Modulus, in y ['e2,']'
call readr(0,ain,nr)
if(nr .eq. 1)then
    e2=ain(1)
endif

e3=11.0e10
write(*,'(a,g12.5,a)')'Enter Young's Modulus, in z ['e3,']'
call readr(0,ain,nr)
if(nr .eq. 1)then
    e3=ain(1)
endif

c    write(*,'(a)')'NOTE. The poisson ratio nuij is the '
c    write(*,'(a)')'the strain ratio -epsilon_j/epsilon_i '
c    write(*,'(a)')'under a normal stress sigma_i'
write(*,*)

c    nu31=.31

write(*,*)'Enter ANSYS Poisson ratios (Theory sec. 2.1)'
prxy=.3
prxz=.41975537
pryz=.41975537
write(*,'(a,g12.5,a)')'Enter Major Poisson ratio prxy['prxy,']'
call readr(0,ain,nr)
if(nr .eq. 1)then
    prxy=ain(1)
endif

write(*,'(a,g12.5,a)')'Enter Major Poisson ratio prxz['prxz,']'
call readr(0,ain,nr)
if(nr .eq. 1)then
    prxz=ain(1)
endif

write(*,'(a,g12.5,a)')'Enter Major Poisson ratio pryz['pryz,']'
call readr(0,ain,nr)
if(nr .eq. 1)then
    pryz=ain(1)
endif
c    g23=(e1*e2)/(e1+e2+2.*nu12*e1)
g23=5.2e10
write(*,'(a,g12.5,a)')'Enter shear modulus g23 ['g23,']'
call readr(0,ain,nr)
if(nr .eq. 1)then
    g23=ain(1)
endif

c    g13=(e1*e2)/(e1+e2+2.*nu12*e1)

```

```

g13=5.2e10
write(*,'(a,g12.5,a)')'Enter shear modulus g13 [' ,g13,']'
call readr(0,ain,nr)
if(nr .eq. 1)then
  g13=ain(1)
endif

c
g12=(e1*e2)/(e1+e2+2.*nu12*e1)
g12=12.2e10
write(*,'(a,g12.5,a)')'Enter shear modulus g12 [' ,g12,']'
call readr(0,ain,nr)
if(nr .eq. 1)then
  g12=ain(1)
endif

c
build the compliance matrix
do i=1,6
  do j=1,6
    cmatinv(i,j)=0.
  enddo
enddo

cmatinv(1,1)=1./e1
cmatinv(2,2)=1./e2
cmatinv(3,3)=1./e3
cmatinv(1,2)=-prxy/e1
cmatinv(2,1)=cmatinv(1,2)
cmatinv(1,3)=-prxz/e1
cmatinv(3,1)=cmatinv(1,3)
cmatinv(2,3)=-pryz/e2
cmatinv(3,2)=cmatinv(2,3)
cmatinv(4,4)=1/g23
cmatinv(5,5)=1/g13
cmatinv(6,6)=1/g12

write(*,*)'Compliance matrix (elasticity inverse) cinv= '
do i=1,6
  write(*,'(6(g11.4,1x))')(cmatinv(i,j),j=1,6)
enddo
write(*,*)'Note: In ANSYS components 4,5,6 are permuted'
write(*,*)'(A different mapping from tensor to vector is used):'
write(*,*)'ANSYS 44 is 66'
write(*,*)'ANSYS 55 is 44'
write(*,*)'ANSYS 66 is 55'
write(*,*)'The five independent compliance coefficients are:'
write(*,'(a,g15.8)')'s11=',cmatinv(1,1)
write(*,'(a,g15.8)')'s33=',cmatinv(3,3)
write(*,'(a,g15.8)')'s44=',cmatinv(4,4)
write(*,'(a,g15.8)')'s12=',cmatinv(1,2)
write(*,'(a,g15.8)')'s13=',cmatinv(1,3)
do i=1,6
  do j=1,6
    a(i,j)=cmatinv(i,j)
  enddo

```

```

enddo
ia=6
ib=6
n=6
m=6
inv=1
eps=1.e-10
idet=0
write(*,*)'Computing inverse matrix'
call gaussr(a,ia,b,ib,n,m,inv,eps,idet,det,ier)
if(ier .ne. 0)then
  write(*,*)' matrix is singular '
endif
do i=1,6
  do j=1,6
    cmat(i,j)=b(i,j)
  enddo
enddo
write(*,*)' Elasticity matrix = '
do i=1,6
  write(*,'(6(g11.4,1x))')(cmat(i,j),j=1,6)
enddo

m=6
n=6
l=6
ic=6
call matm(cmatinv,ia,m,n,b,ib,l,cc,ic)
c  write(*,*)'Compliance matrix times elasticity matrix= '
c  do i=1,6
c    write(*,'(6(g11.4,1x))')(cc(i,j),j=1,6)
c  enddo
endif
c  end of case1

c  begin case2
if(reply .eq. '2')then
do i=1,6
do j=1,6
  cmat(i,j)=0.
enddo
enddo

cmat(1,1)=13.2e10
write(*,'(a,g12.5,a)')'Enter c11 [' ,cmat(1,1),' ]'
call readr(0,ain,nr)
if(nr .eq. 1)then
  cmat(1,1)=ain(1)
endif

cmat(2,2)=cmat(1,1)

cmat(3,3)=11.5e10
write(*,'(a,g12.5,a)')'Enter c33 [' ,cmat(3,3),' ]'

```

```

call readr(0,ain,nr)
if(nr .eq. 1)then
  cmat(3,3)=ain(1)
endif

cmat(4,4)=5.2e10
write(*,'(a,g12.5,a)')'Enter c44 (ANSYS c66) [' ,cmat(4,4),']'
call readr(0,ain,nr)
if(nr .eq. 1)then
  cmat(4,4)=ain(1)
endif
cmat(5,5)=cmat(4,4)
cmat(1,2)=7.1e10
write(*,'(a,g12.5,a)')'Enter c12 [' ,cmat(1,2),']'
call readr(0,ain,nr)
if(nr .eq. 1)then
  cmat(1,2)=ain(1)
endif
cmat(2,1)=cmat(1,2)

  cmat(1,3)=7.3e10
write(*,'(a,g12.5,a)')'Enter c13 [' ,cmat(1,3),']'
call readr(0,ain,nr)
if(nr .eq. 1)then
  cmat(1,3)=ain(1)
endif

cmat(3,1)=cmat(1,3)
cmat(2,3)=cmat(1,3)
cmat(3,2)=cmat(2,3)
cmat(6,6)=2.*(cmat(1,1)-cmat(1,2))

write(*,*)'Elasticity matrix c='
do i=1,6
  write(*,'(6(g11.4,1x))')(cmat(i,j),j=1,6)
enddo
write(*,*)'Note: In ANSYS components 4,5,6 are permuted'
write(*,*)'ANSYS 44 is 66'
write(*,*)'ANSYS 55 is 44'
write(*,*)'ANSYS 66 is 55'

do i=1,6
  do j=1,6
    a(i,j)=cmat(i,j)
  enddo
enddo

ia=6
ib=6
n=6
m=6
inv=1
eps=1.e-10
idet=0

```

```

call gaussr(a,ia,b,ib,n,m,inv,eps,idet,det,ier)
if(ier .ne. 0)then
  write(*,*)' matrix is singular '
endif

do i=1,6
  do j=1,6
    cmatinv(i,j)=b(i,j)
  enddo
enddo

write(*,*)'Compliance matrix= '
do i=1,6
  write(*,'(6(g11.4,1x))')(cmatinv(i,j),j=1,6)
enddo
m=6
n=6
l=6
ic=6
call matm(cmat,ia,m,n,cmatinv,ib,l,cc,ic)
c   write(*,*)'Compliance matrix times elasticity matrix= '
c   do i=1,6
c     write(*,'(6(g11.4,1x))')(cc(i,j),j=1,6)
c   enddo
endif
c   end case2

c   begin case3
c   if(reply .eq. '3')then

do i=1,6
  do j=1,6
    cmatinv(i,j)=0.
  enddo
enddo

cmatinv(1,1) = 1.65e-11
cmatinv(3,3) = 2.07e-11
cmatinv(4,4) = 4.35e-11
cmatinv(1,2) = -4.78e-12
cmatinv(1,3) = -8.45e-12

write(*,'(a,g12.5,a)')'Enter s11 [' ,cmatinv(1,1),']'
write(*,*)'Reference for default values:'
write(*,*)'Auld, Appendix A.4, PZT-5H'
call readr(0,ain,nr)
if(nr .eq. 1)then
  cmatinv(1,1)=ain(1)
endif

cmatinv(2,2)=cmatinv(1,1)

```

```

write(*,'(a,g12.5,a)')'Enter s33 [' ,cmatinv(3,3),']'
call readr(0,ain,nr)
if(nr .eq. 1)then
  cmatinv(3,3)=ain(1)
endif

write(*,'(a,g12.5,a)')'Enter s44 (ANSYS c66) [' ,cmatinv(4,4),']'
call readr(0,ain,nr)
if(nr .eq. 1)then
  cmatinv(4,4)=ain(1)
endif
cmatinv(5,5)=cmatinv(4,4)
write(*,'(a,g12.5,a)')'Enter s12 [' ,cmatinv(1,2),']'
call readr(0,ain,nr)
if(nr .eq. 1)then
  cmatinv(1,2)=ain(1)
endif
cmatinv(2,1)=cmatinv(1,2)

write(*,'(a,g12.5,a)')'Enter s13 [' ,cmatinv(1,3),']'
call readr(0,ain,nr)
if(nr .eq. 1)then
  cmatinv(1,3)=ain(1)
endif

cmatinv(3,1)=cmatinv(1,3)
cmatinv(2,3)=cmatinv(1,3)
cmatinv(3,2)=cmatinv(2,3)
cmatinv(6,6)=2.*(cmatinv(1,1)-cmatinv(1,2))

write(*,*)'Compliance matrix s= cinv ='
do i=1,6
  write(*,'(6(g11.4,1x)')')(cmatinv(i,j),j=1,6)
enddo
write(*,*)'Note: In ANSYS components 4,5,6 are permuted'
write(*,*)'ANSYS 44 is 66'
write(*,*)'ANSYS 55 is 44'
write(*,*)'ANSYS 66 is 55'

do i=1,6
  do j=1,6
    a(i,j)=cmatinv(i,j)
  enddo
enddo
ia=6
ib=6
n=6
m=6
inv=1
eps=1.e-10
idet=0
write(*,*)'Computing inverse matrix'
call gaussr(a,ia,b,ib,n,m,inv,eps,idet,det,ier)
if(ier .ne. 0)then
  write(*,*)' matrix is singular '

```

```

endif
do i=1,6
  do j=1,6
    cmat(i,j)=b(i,j)
  enddo
enddo
write(*,*)' Elasticity matrix = '
do i=1,6
  write(*,'(6(g11.4,1x))')(cmat(i,j),j=1,6)
enddo

m=6
n=6
l=6
ic=6
call matm(cmatinv,ia,m,n,b,ib,l,cc,ic)
c write(*,*)'Compliance matrix times elasticity matrix= '
c do i=1,6
c   write(*,'(6(g11.4,1x))')(cc(i,j),j=1,6)
c enddo

endif
c end case3
write(*,*)'Orthotropic constants from compliance matrix:'
e1=1./cmatinv(1,1)
e2=1./cmatinv(2,2)
e3=1./cmatinv(3,3)
write(*,'(a,g15.8)')'Young's Modulus 1= ',e1
write(*,'(a,g15.8)')'Young's Modulus 2= ',e2
write(*,'(a,g15.8)')'Young's Modulus 3= ',e3

prxy=-cmatinv(1,2)*e1
write(*,'(a,g15.8)')'Major Poisson ratio prxy=',prxy
prxz=-cmatinv(1,3)*e1
write(*,'(a,g15.8)')'Major Poisson ratio prxz=',prxz
pryz=-cmatinv(2,3)*e2
write(*,'(a,g15.8)')'Major Poisson ratio pryz=',pryz

g23= 1./cmatinv(4,4)
g13= 1./cmatinv(5,5)
g12= 1./cmatinv(6,6)
write(*,'(a,g15.8)')'Shear Modulus g23= ',g23
write(*,'(a,g15.8)')'Shear Modulus g13= ',g13
write(*,'(a,g15.8)')'Shear Modulus g12= ',g12
write(*,*)
c orthotropic elasticity tensor
do i=1,3
  do j=1,3
    do k=1,3
      e(i,j,k)=0.
      do l=1,3
        c(i,j,k,l)=0.
      enddo
    enddo
  enddo
enddo

```

```

        enddo
        do i=1,3
            do j=1,3
                call t2vec(i,j,ij)
                do k=1,3
                    do l=1,3
                        call t2vec(k,l,kl)
                        if(k .eq. l)then
                            c(i,j,k,l)=cmat(ij,kl)
                        else
                            c(i,j,k,l)=cmat(ij,kl)/2.
                        endif
                    enddo
                enddo
            enddo
        enddo
c
        write(*,*)'Do you want to see the nonzero components'
        write(*,*)'of the 81 elasticity tensor components of c? [n]'
        read(*,'(a)')reply
        if(lenstr(reply) .eq. 0)then
            reply='n'
        endif
        if(reply .eq. 'y')then
            do i=1,3
                do j=1,3
                    do k=1,3
                        do l=1,3
                            if(c(i,j,k,l) .ne. zero)then
                                write(*,'(a,i1,i1,i1,i1,a,g15.8)')'c(',i,j,k,l,')=',c(i,j,k,l)
                            endif
                        enddo
                    enddo
                enddo
            enddo
            write(*,*)'piezoelectric (stress) tensor coefficients'
            do i=1,3
                do j=1,3
                    do k=1,3
                        e(i,j,k)=0.
                        do l=1,3
                            do m=1,3
                                e(i,j,k) = e(i,j,k) + d(i,l,m)*c(l,m,j,k)
                            enddo
                        enddo
                    enddo
                enddo
            enddo
c
            write(*,'(a,i1,i1,i1,a,g15.8)')'e(',i,j,k,')=',e(i,j,k)
            enddo
            enddo
            write(*,*)'Do you want to see the nonzero components'
            write(*,*)'of the 27 piezoelectric tensor components of e? [n]'
            read(*,'(a)')reply
            if(lenstr(reply) .eq. 0)then
                reply='n'
            endif
        endif
    enddo
enddo

```

```

endif
if(reply .eq. 'y')then
  do i=1,3
    do j=1,3
      do k=1,3
        if(e(i,j,k) .ne. zero)then
          write(*,'(a,i1,i1,i1,a,g15.8)')'e(',i,j,k,')=' ,e(i,j,k)
        endif
      enddo
    enddo
  enddo
endif

c   compute the e piezoelectric matrix

do i=1,3
  do jk=1,6
    call vec2t(jk,j,k)
    if(j .eq. k)then
      emat(jk,i) = e(i,j,k)
    else
      emat(jk,i) = 2.*e(i,j,k)
    endif
  enddo
enddo

c

write(*,*)'Piezoelectric e matrix= '
do i=1,6
  write(*,'(3(g11.4,1x))')(emat(i,j),j=1,3)
enddo
write(*,*)'Second calculation:'
e13=cmat(1,1)*dmat(3,1)+cmat(1,2)*dmat(3,1)+cmat(1,3)*dmat(3,3)
write(*,*)'e13=',e13
e33=cmat(1,3)*dmat(3,1)+cmat(1,3)*dmat(3,1)+cmat(3,3)*dmat(3,3)
write(*,*)'e33=',e33
e42=cmat(4,4)*dmat(1,5)
write(*,*)'e42=',e42

write(*,*)'Note: ANSYS e rows are permuted'
write(*,*)'ANSYS row 4 is row 6 '
write(*,*)'ANSYS row 5 is row 4 '
write(*,*)'ANSYS row 6 is row 5 '

write(*,*)'ANSYS Piezoelectric e matrix= '
do i=1,3
  write(*,'(3(g11.4,1x))')(emat(i,j),j=1,3)
enddo
write(*,'(3(g11.4,1x))')(emat(6,j),j=1,3)
write(*,'(3(g11.4,1x))')(emat(4,j),j=1,3)
write(*,'(3(g11.4,1x))')(emat(5,j),j=1,3)

c   dielectric constants (farads/meter)

```

```

c      dc(1,1)=7.12e-9
c      dc(2,2)=7.12e-9
c      dc(3,3)=5.84e-9
c      density (kilograms/meter^3)
c      density=7.5e3
cm=', '
idmat=3
m=3
n=6
iemat=6
l=3
icc=6
call matm(dmat,idmat,m,n,emat,iemat,l,cc,icc)
write(*,*)'d times e = '
do i=1,3
  write(*,'(3(g11.4,1x))')(cc(i,j),j=1,3)
enddo

c      compute permittivity tensor
eps0=8.85e-12
do i=1,3
  do j=1,3
    epsilon(i,j)=0.
  enddo
enddo
dielx=1300.
write(*,'(a)')'Enter the unclamped dielectric constant K11'
write(*,'(a,g15.8,a)')[',dielx,']'
call readr(0,ain,nr)
if(nr .eq. 1)then
  dielx=ain(1)
endif
epsilon(1,1)=dielx*eps0

diely=dielx
c      cs1='Enter the dielectric constant K22'
c      write(*,'(a,a,g15.8,a)')cs1, [' ,diely,']'
c      call readr(0,ain,nr)
c      if(nr .eq. 1)then
c        diely=ain(1)
c      endif
epsilon(2,2)=diely*eps0

dielz=1000.
write(*,'(a)')'Enter the unclamped dielectric constant K33'
write(*,'(a,g15.8,a)')[',dielz,']'
call readr(0,ain,nr)
if(nr .eq. 1)then
  dielz=ain(1)
endif
epsilon(3,3)=dielz*eps0

write(*,*)'Unclamped electric permittivity tensor='
do i=1,3

```

```

write(*,'(3(g11.4,1x))')(epsilon(i,j),j=1,3)
enddo

do i=1,3
do j=1,3
clamped(i,j)= epsilon(i,j) - cc(i,j)
enddo
enddo

write(*,*)'Clamped electric permittivity tensor='
do i=1,3
write(*,'(3(g11.4,1x))')(clamped(i,j),j=1,3)
enddo

do i=1,3
do j=1,3
sum=0.
do k=1,3
do l=1,3
sum=sum+d(i,k,l)*e(j,k,l)
c if((i .eq. 1) .and. (j .eq. 1))then
c write(*,*)k,l,' d= ',d(i,k,l),' e= ',e(j,k,l)
c endif
enddo
enddo
epsilon(i,j)=epsilon(i,j)-sum
enddo
enddo
c write(*,*)'Clamped electric permittivity tensor='
c do i=1,3
c write(*,'(3(g11.4,1x))')(epsilon(i,j),j=1,3)
c enddo
rho= 7.500000E-03

write(*,*)'Enter density Kg/m^3 [7500]'
call readr(0,ain,nr)
if(nr .ge. 1)then
rho=ain(1)
endif

write(*,'(a)')'/COM '
write(*,'(a)')'/COM Material properties'
c material number
mn=2
write(*,'(a)')'/COM Young's Moduli'
write(*,'(a,i3,a,g15.8)')'EX',mn,cm,e1
write(*,'(a,i3,a,g15.8)')'EY',mn,cm,e2
write(*,'(a,i3,a,g15.8)')'EZ',mn,cm,e3
write(*,'(a)')'/COM ANSYS "Major" Poisson ratios'
c reference ANSYS Theory 2.1 structural fundamentals
prxy=-cmatinv(1,2)*e1
write(*,'(a,i3,a,g15.8)')'PRXY',mn,cm,prxy

```

```

prxz=-cmatinv(1,3)*e1
write(*,'(a,i3,a,g15.8)')'PRXZ,' ,mn,cm,prxz
pryz=-cmatinv(2,3)*e2
write(*,'(a,i3,a,g15.8)')'PRYZ,' ,mn,cm,pryz
write(*,'(a)')'/COM Density Kg/m^3'
write(*,'(a,i3,a,g15.8)')'/COM DENS,' ,mn,cm,rho
write(*,'(a)')'/COM clamped Permittivities'

c   epsilon(1,1)=8.7969e-12
c   epsilon(2,2)=8.7969e-12
c   epsilon(3,3)=8.7969e-12
write(*,'(a,i3,a,g15.8)')'MP,PERX,' ,mn,cm,clamped(1,1)
write(*,'(a,i3,a,g15.8)')'MP,PERY,' ,mn,cm,clamped(2,2)
write(*,'(a,i3,a,g15.8)')'MP,PERZ,' ,mn,cm,clamped(3,3)
write(*,'(a)')'/COM ANSYS Piezoelectric "e" matrix'
write(*,'(a)')'TB,PIEZ,3'
write(*,'(a,i3,a,g15.8)')'TBDATA,' ,3,cm,emat(1,3)
write(*,'(a,i3,a,g15.8)')'TBDATA,' ,6,cm,emat(2,3)
write(*,'(a,i3,a,g15.8)')'TBDATA,' ,9,cm,emat(3,3)
c   ansysemat(5,2)=emat(4,2)
write(*,'(a,i3,a,g15.8)')'TBDATA,' ,14,cm,emat(4,2)
c   ansysemat(6,1)=emat(5,1)
write(*,'(a,i3,a,g15.8)')'TBDATA,' ,16,cm,emat(5,1)
end

c+ readr read a row of floating point numbers
subroutine readr(nf, a, nr)
implicit real*8(a-h,o-z)
c   numbers are separated by spaces
c   examples of valid numbers are:
c   12.13 34 45e4 4.78e-6 4e2,5.6D-23,10000.d015
c   nf=file number, 0 for standard input file
c   a=array of returned numbers
c   nr=number of values in returned array,
c   or 0 for empty or blank line,
c   or -1 for end of file on unit nf.
c requires functions val and length
dimension a(*)
character*200 b
character*200 c
character*1 d
c=' '
if(nf.eq.0)then
read(*,'(a)',end=99)b
else
read(nf,'(a)',end=99)b
endif
nr=0
l=lenstr(b)
if(l.ge.200)then
write(*,*)' error in readr subroutine '
write(*,*)' record is too long '
endif
do 1 i=1,l
d=b(i:i)
if (d.ne.' ') then

```

```

        k=lenstr(c)
        if (k.gt.0)then
            c=c(1:k)//d
        else
            c=d
        endif
    endif
    if( (d.eq.' ').or.(i.eq.1)) then
        if (c.ne.' ') then
            nr=nr+1
            call valsub(c,a(nr),ier)
            c=' '
        endif
    endif
1    continue
    return
99   nr=-1
    return
    end

c
c+ lenstr  nonblank length of string
    function lenstr(s)
c  length of the substring of s obtained by deleting all
c  trailing blanks from s.  thus the length of a string
c  containing only blanks will be 0.
    character  s*(*)
    lenstr=0
    n=len(s)
    do 10 i=n,1,-1
        if(s(i:i) .ne. ' ')then
            lenstr=i
            return
        endif
10   continue
    return
    end

c+ valsub converts string to floating point number (r*8)
    subroutine valsub(s,v,ier)
    implicit real*8(a-h,o-z)
c  examples of valid strings are: 12.13 34 45e4 4.78e-6 4E2
c  the string is checked for valid characters,
c  but the string can still be invalid.
c  s-string
c  v-returned value
c  ier- 0 normal
c       1 if invalid character found, v returned 0
c
    logical p
    character s*(*),c*50,t*50,ch*15
    character z*1
    data ch/'1234567890+-.eE'/
    v=0.
    ier=1
    l=lenstr(s)
    if(l.eq.0)return

```

```

p=.true.
do 10 i=1,1
z=s(i:i)
if((z.eq.'D').or.(z.eq.'d'))then
s(i:i)='e'
endif
p=p.and.(index(ch,s(i:i)).ne.0)
10 continue
if(.not.p)return
n=index(s,'.')
if(n.eq.0)then
n=index(s,'e')
if(n.eq.0)n=index(s,'E')
if(n.eq.0)n=index(s,'d')
if(n.eq.0)n=index(s,'D')
if(n.eq.0)then
s=s(1:1)//'.'
else
t=s(n:1)
s=s(1:(n-1))//'.'//t
endif
endif
l=l+1
endif
write(c,'(a30)')s(1:1)
read(c,'(g30.23)')v
ier=0
return
end
c+ str floating point number to string
subroutine str(x,s)
implicit real*8(a-h,o-z)
character s*25,c*25,b*25,e*25
zero=0.
if(x.eq.zero)then
s='0'
return
endif
write(c,'(g11.4)')x
read(c,'(a25)')b
l=lenstr(b)
do 10 i=1,1
n1=i
if(b(i:i).ne.' ')go to 20
10 continue
20 continue
if(b(n1:n1).eq.'0')n1=n1+1
b=b(n1:1)
l=l+1-n1
k=index(b,'E')
if(k.gt.0)e=b(k:1)
if(k.gt.0)then
s=b(1:(k-1))
k1=index(b,'E+0')
if(k1.gt.0)then
e='E'//b((k1+3):1)

```

```

        else
            k1=index(b,'E+')
            if(k1.gt.0)e='E'//b((k1+2):1)
        endif
        k1=index(b,'E-0')
        if(k1.gt.0)e='E-'//b((k1+3):1)
        l=k-1
    else
        s=b
    endif
    j=index(s, '.')
    n2=1
    if(j.ne.0)then
        do 30 i=1,l
            n2=1+1-i
            if(s(n2:n2).ne.'0')go to 40
30         continue
        endif
40         continue
        s=s(1:n2)
        if(s(n2:n2).eq.'.')then
            s=s(1:(n2-1))
            n2=n2-1
        endif
        if(k.gt.0)s=s(1:n2)//e
    return
end
c+ t2vec stress tensor index to vector index
subroutine t2vec(i,j,k)
c input:
c i,j
c output:
c k
c so that sigma(k)=sigma(i,j)
integer m(3,3)
data m/1,6,5,6,2,4,5,4,3/
k=m(i,j)
return
end
c+ vec2t stress vector index to tensor index
subroutine vec2t(k,i,j)
c input:
c k
c output:
c i,j
c so that sigma(i,j)=sigma(k)
integer mi(6),mj(6)
data mi/1,2,3,2,1,1/
data mj/1,2,3,3,3,2/
i=mi(k)
j=mj(k)
return
end
c+ gaussr solution of linear equations, inverse, determinant (real) new version
c gaussr2.ftn 4/17/96 modernization of gaussr under construction

```

```

      subroutine gaussr(a,ia,b,ib,n,m,inv,eps,idet,det,ier)
c solves the equation a*c=b for c, where a is an n by n matrix
c c and b are n row by m column matrices. c is returned as b.
c algorithm      -gaussian elimination with partial pivoting.
c parameters     a-n by n matrix containing the coefficients of
c                the linear system.
c                ia-row dimension of a in defining routine
c                e.g. in the routine where a is defined,
c                a might be dimensioned as:
c                dimension a(nr,nc)
c                then ia must be set to nr. we may have n < ia, but
c                must not have n > ia, or n*n > nr*nc.
c                ia is needed for proper addressing of matrix a.
c                fortran stores by column first: a(i,j)=a(i+(j-1)*ia))
c                b-n by m matrix containing the m right sides
c                of the equations, on entering. on returning, b
c                contains the solutions. the inverse of a is
c                returned in b when inv=1
c                ib-row dimension of b in defining routine
c                n-row and column dimension of a.
c                m-column dimension of b (usually 1)
c                the program changes m to n when inv=1
c inv-the inverse of a is calculated
c and returned in b when inv=1
c b must be large enough to hold the inverse
c eps-each equation is normalized so that the
c coefficients are <= 1 in magnitude.
c when a pivot is less than eps the matrix is
c considered nearly singular, and ier is set to 1
c if a pivot is zero the matrix is singular, and
c ier is set to 2. one may set eps=1.e-5 for
c single precision, and 1.e-12 for double.
c eps does not effect any calculation.
c normalization may also prevent exponent overflow.
c idet-compute determinant only if idet = 1
c determinants are products of n numbers.
c overflow can occur if the elements of the
c matrix have large exponents.
c set idet=0 if the determinant is not needed.
c det-determinant of a.
c ier-return parameter,
c ier=0 normal return
c ier=1 matrix is nearly singular
c ier=2 matrix is singular
c
c warning!! the subroutine changes a and b. if they need to be
c saved, copies must be made before calling the subroutine.
c the subroutine can be converted to different number type
c by uncommenting the appropriate implicit statement.
      implicit real*8(a-h,o-z)
c      implicit complex(a-h,o-z)
c      implicit complex*16(a-h,o-z)
      logical cdet
      dimension a(ia,*),b(ib,*)
      cdet = idet .eq. 1

```

```

zero=0.
ier=0
det=1.
if(m.le.0)m=1
if(inv.eq.1)then
c   set b equal to the identity
   do i=1,n
     do j=1,n
       b(i,j)=0.
       if(i.eq.j)b(i,j)=1.
     enddo
   enddo
   m=n
endif
c normalize rows
do 20 i=1,n
  biggest=a(i,1)
  do 16 j=2,n
    ab=a(i,j)
    if(abs(ab).gt.abs(biggest))bigest=ab
16  continue
    if(bigest.eq.zero)then
      ier=2
      det=0.
      return
    endif
    if(cdet)det=det*bigest
    do 18 j=1,n
      a(i,j)=a(i,j)/bigest
18  continue
      do 19 j=1,m
        b(i,j)=b(i,j)/bigest
19  continue
20  continue
    j=1
    do while( j .lt. n)
      kk=j+1
      l=j
c   find row l with largest pivot
      do 32 i=kk,n
        if(abs(a(i,j)).gt.abs(a(l,j)))l=i
32  continue
        if(abs(a(l,j)).eq.zero)then
          ier=2
          det=0.
          return
        endif
        if(abs(a(l,j)).le.abs(eps))ier=1
        if(l.ne.j)then
c   interchange rows l and j
          do 37 k=1,n
            c=a(l,k)
            a(l,k)=a(j,k)
            a(j,k)=c
37  continue

```

```

do 39 k=1,m
  c=b(1,k)
  b(1,k)=b(j,k)
  b(j,k)=c
39  continue
  if(cdet)det=det*(-1.)
endif
if(cdet)det=det*a(j,j)
c divide row by pivot
c=a(j,j)
do 50 k=j,n
  a(j,k)=a(j,k)/c
50  continue
do 55 k=1,m
  b(j,k)=b(j,k)/c
55  continue
c add multiple of row j to lower rows
c to eliminate jth coefficients
jj=j+1
do 80 i=jj,n
  am=a(i,j)
  do 60 k=1,n
    a(i,k)=a(i,k)-am*a(j,k)
60  continue
do 70 k=1,m
  b(i,k)=b(i,k)-am*b(j,k)
70  continue
80  continue
j=j+1
enddo
am=a(n,n)
if(abs(am).eq.zero)then
  ier=2
  det=0.
  return
endif
if(abs(am).le.abs(eps))ier=1
if(cdet)det=det*am
c a is now in triangular form
c compute nth component of solution
do 90 k=1,m
  b(n,k)=b(n,k)/am
90  continue
c back substitute to compute n-i component
c i=1,2,3,...
nn=n-1
do 120 i=1,nn
  ni=n-i
  do 110 j=1,m
    nj=ni+1
    do 100 ki=nj,n
      b(ni,j)=b(ni,j)-a(ni,ki)*b(ki,j)
100  continue
110  continue
120  continue

```

```

        return
    end
    subroutine matm(a,ia,m,n,b,ib,l,c,ic)
        implicit real*8(a-h,o-z)
c arguments
c a-matrix
c ia-row dimension of a in calling program
c m-number of rows of a
c n-number of columns of a
c b-matrix
c ib-row dimension of b in calling program
c l-number of columns of b
c c-product matrix: c=a*b
c ic-row dimension of c in calling program
c
        dimension a(ia,*),b(ib,*),c(ic,*)
c      c=a*b
        do 10 i=1,m
            do 10 j=1,l
                c(i,j)=0.
                do 5 k=1,n
5          c(i,j)=c(i,j)+a(i,k)*b(k,j)
10        continue
            return
        end

```

0.22 Location of Piezoelectric Information In ANSYS Manuals

- Index assignment for stress-strain tensor to six-vector: Theory 2.1.
- Major and Minor Poisson ratios, orthotropic compliance matrices: Theory 2.1.
- Piezoelectric Solid Element SOLID5: Elements manual 4-39.
- Piezoelectric Plane Element PLANE13: Elements manual 4-85.
- Piezoelectric Tetrahedral Coupled Solid Element SOLID98: Elements manual 4-653.
- Piezoelectric Analysis: Procedures Coupled-Field Analysis 8.1, piezoelectric analysis p8-13 to p8-15.
- Defining matrices: Commands TB and TBDATA, Commands manual.

- Piezoelectrics Theory: Theory Manual 11.1 to page 11-17.
- Example VM175: Natural Frequency of a Piezoelectric Transducer ... (ANSYS Stuff Notebook). Reference: Boucher D, Lagier M, Maerfeld C, IEEE Transactions on Sonics and Ultrasonics, Volume SU-28, No. 5 September 1981 pp318-330.
- Example VM176: Frequency Response of Electrical Admittance for a Piezoelectric Transducer, Reference: Kagawa and Yamabuchi, IEEE Trans. Sonics and Ultrasonics, V. SU-26, No. 2, March 1979.
- Example PM13, ANSYS Examples Supplement, Piezoelectric Beam Resonator

0.23 ANSYS Comparison For A Composite Piezoelectric Transducer: Example VM176.

Reference for the problem: Kagawa and Yamabuchi **Finite Element Simulation of a Composite Piezoelectric Transducer** IEEE Transactions on Sonics and Ultrasonics, Vol. SU-26, No 2 march 1979.

The transducer consists of cylindrical disks of aluminum, adhesive, PZT, adhesive, and aluminum.

PZT material: NEPEC 6

Elasticity matrix

$$c = 10^{10} \begin{bmatrix} 12.8 & 6.8 & 6.6 & 0 & 0 & 0 \\ 6.8 & 12.8 & 6.6 & 0 & 0 & 0 \\ 6.6 & 6.6 & 11 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2.1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2.1 \end{bmatrix}$$

Piezoelectric matrix (*Coulomb/Meter²*)

$$e = \begin{bmatrix} 0 & 0 & -6.1 \\ 0 & 0 & -6.1 \\ 0 & 0 & 15.7 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Dielectric Matrix (*Farad/Meter*)

$$\epsilon = 10^{-9} \begin{bmatrix} 8.7969 & 0 & 0 \\ 0 & 8.7969 & 0 \\ 0 & 0 & 8.7969 \end{bmatrix}$$

See ANSYS notebook for more details.

0.24 Piezoelectricity Bibliography

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