A Sample

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12/5/2008

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1 Preface

This is a small LaTeX file to be used in demonstrating MikTeX.

2 Rolles' Theorem

Let f be a nice function. Suppose $f(a) = f(b) = 0$. Then there is a number c, $a < c < b$ so that $f'(c) = 0$.
Proof. There must be a relative maximum or a relative minimum of f at some point c between a and b.

3 The Mean Value Theorem

Let f be a nice function. There exists a point c, $a < c < b$ so that

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Proof. Let function g be formed by subtracting the straight line that passes through the points $(a, f(a))$ and $(b, f(b)$. Then

$$g(x) = f(x) - [\frac{b - x}{b - a} f(a) + \frac{a - x}{a - b} f(b)].$$

Then $g(a) = g(b) = 0$, so by Rolles's Theorem there is a c, $a < c < b$, so that $g'(c) = 0$. Then

$$0 = g'(c) = f'(c) - \frac{f(b) - f(a)}{b - a}.$$

Hence

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

4 Taylor's Formula

If we apply Rolles' theorem repeatedly, we get an extension of the mean value theorem, which is called Taylor's formula. And from that the Taylor series expansion of a function.
Theorem *Taylor's Formula* If f is a nice function (has continuous derivatives), then there is a number c, $a < c < b$, so that

$$f(b) = f(a) + f^{(1)}(a)(x - a) + f^{(2)}(a)\frac{(x - a)^2}{2!} + f^{(3)}(a)\frac{(x - a)^3}{3!} + ... +$$

$$f^{(n-1)}(a)\frac{(x - a)^{n-1}}{(n - 1)!} + f^{(n)}(c)\frac{(x - a)^n}{n!}$$

Proof. Let p be the polynomial

$$p(x) = f(a) + f^{(1)}(a)(x - a) + f^{(2)}(a)\frac{(x - a)^2}{2!} + ... + f^{(n-1)}(a)\frac{(x - a)^{n-1}}{(n - 1)!}.$$

Let M be defined by

$$f(b) = p(b) + \frac{M(b-a)^n}{n!}.$$

Define

$$g(x) = f(x) - (p(x) + \frac{M(x-a)^n}{n!})$$

. Then $g(a) = f(a) - f(a) + 0 = 0$, and $g(b) = 0$ by the definition of M. By Rolles' Theorem there is a number b_1, $a < b_1 < b$, so that $g'(b_1) = 0$. Clearly $g'(a) = 0$, so we may apply Rolles' Theorem to g' and obtain a number $b_2, a < b_2 < b_1$, so that $g^{(2)}(b_2) = 0$. Continuing in this way, after n steps, we find that there is a c so that $a < c < b$ and

$$f^{(n)}(c) - M = g^{(n)}(c) = 0.$$

Therefore $M = f^{(n)}(c)$ and

$$f(b) = p(b) + f^{(n)}(c)\frac{(b-a)^n}{n!},$$

which is Taylor's Formula.

So the Taylor series for $\sin(x)$ is

$$\sin(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ...$$

and for $\cos(x)$ is

$$\cos(x) = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + ...$$

The complex Taylor series for $\exp(z)$ about zero is

$$\exp(z) = \sum_{n=0}^{\infty} \frac{z^n}{n!} = 1 + \frac{z}{1!} + \frac{z^2}{2!} + \frac{z^3}{3!} + ...$$

So

$$e^{i\theta} = \exp(i\theta) = 1 + i\frac{\theta}{1!} + (i)^2\frac{\theta^2}{2!} + (i)^3\frac{\theta^3}{3!} + (i)^4\frac{\theta^4}{3!} + (i)^5\frac{\theta^5}{5!} + ...$$

$$= 1 + i\frac{\theta}{1!} - \frac{\theta^2}{2!} - i\frac{\theta^3}{3!} + \frac{\theta^4}{4!} + i\frac{\theta^5}{5!} - \frac{\theta^6}{6!} - i\frac{\theta^7}{7!} + ...$$

$$= \cos(\theta) + i\sin(\theta).$$

Thus if $z = x + iy$ and

$$\arg(z) = \theta,$$

Then

$$z = |z|e^{i\theta},$$

and

$$z_1 z_2 = |z_1||z_2|e^{i(\theta_1+\theta_2)}.$$

5 The Laplace Transform

$$L(f)(s) = \int_0^\infty f(t)e^{-st}dt,$$

$$\int_0^\infty \cos(kt)e^{-st}dt = \frac{s}{s^2+k^2},$$

$$\int_0^\infty \sin(kt)e^{-st}dt = \frac{k}{s^2+k^2}$$

6 Matrix

If **A** is the matrix

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix},$$

then its eigenvalues λ are the roots of the equation

$$\begin{vmatrix} a_{11}-\lambda & a_{12} & a_{13} \\ a_{21} & a_{22}-\lambda & a_{23} \\ a_{31} & a_{32} & a_{33}-\lambda \end{vmatrix} = 0.$$

7 Print It Verbatim

If we want to include material that is not to be typeset, we may enclose it between a begin, and an end, verbatim tag.

```
\[ {\bf A} =
\left[
\begin{array}{ccc}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{array}
\right] ,
\]
```

For some

$$\tau\epsilon\chi$$

is a big thing, but for others

$$\scriptstyle \tau\epsilon\chi$$

is quite small.