Static Electricity Machines

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9/25/2014 Edition

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1 Introduction

Static electricity machines are a staple of the physics demonstration racket. They amaze and are always quite popular. However, they are also good examples of the limiting nature of demonstrations for education. They are limiting because one often does not really get much understanding of the underlying science from them. Looking at the sun is quite impressive, but it does not lead to an understanding of the nuclear reaction processes going on inside. So staring at the sun may not produce much scientific illumination, but it may likely produce extreme over-illumination of the retna, and might well lead to blindness. So maybe we can gain some better understanding of these great spark making machines without going blind.

2 Coulomb's Law

Given two electric point charges q_1 and q_2, there is an electric force between them of magnitude

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2},$$

where r is the distance between them, and where the force is directed along a line passing through the two charges. This is Coulomb's law.

Let n charges q_i be placed at positions $\mathbf{r'}_i$. Let $\mathbf{a} = \mathbf{r} - \mathbf{r'}$., Then a charge q at position vector $\mathbf{r}$ experiences a Coulomb force caused by the n charges given by

$$\mathbf{F}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{qq_i\mathbf{a}_i}{a_i^3}.$$

The force per unit charge is

$$\frac{\mathbf{F}(\mathbf{r})}{q} = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i\mathbf{a}_i}{a_i^3},$$

This gives a field of force that varies with position $\mathbf{r}$, and is caused by the sum of the n charges. This is called the electric field

$$\mathbf{E} = \frac{\mathbf{F}(\mathbf{r})}{q}.$$

If a charge is moved from one position to another, work will be gained or lost. This is potential energy. The potential difference between two points in space is this potential energy change divided by the charge q. That is, the potential difference is the difference in potential energy per unit charge. So a collection of charges gives rise to a field of force, and an electrical potential field.

3 Maxwell's Equations

The Maxwell Equations in MKS form are

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t},$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \cdot \mathbf{D} = \rho,$$

$$\nabla \cdot \mathbf{B} = 0.$$

In a vacuum

$$\mathbf{B} = \mu_0\mathbf{H},$$

$$\mathbf{D} = \epsilon_0\mathbf{E}.$$

So in particular

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0},$$

which is implied by Coulomb's law.

4 Potential

Because

$$\nabla \times \mathbf{E} = 0,$$

a line integral of $\mathbf{E}$ is independent of the path and there exists a potential ϕ so that

$$\mathbf{E} = -\nabla \phi.$$

This follows from Stokes law. See Vector Anakysis. We have

$$\phi = \int \mathbf{E} \cdot d\mathbf{l}.$$

For a point charge at the origin of the coordinate system, the potential at distance r from the origin is

$$\phi(r) = \frac{1}{4\pi\epsilon_0} \frac{q}{r}.$$

The potential due to a set of n point charges is the sum of the n individual potientials.

5 Gauss's Law

Let S be a sphere. Let q be a point charge at the center of S. Then

$$\int_S \mathbf{E} \cdot \mathbf{n} ds = q/\epsilon_0$$

Let S be surrounded by an arbitrary surface G. Integrating the volume bounded by S and G we deduce that that the integral over G equals the integral over S. The integral of $\mathbf{E}$ over the surface of a volume not containing sources is zero. This follows because in such a volume

$$\nabla \cdot \mathbf{E} = 0$$

We conclude that the integral of a field $\mathbf{E}$ over a surface G, which is due to point charges, is equal to the sum of the point charges contained within the surface, divided by the permitivity of free space.

Referring to the figure, by similarity of the triangles, the area element ds' and the projection of ds, which is $ds\cos(\theta)$, have the ratio

$$\frac{\hat{r}\cdot ds}{ds'} = r^2.$$

So

$$\frac{\hat{r}\cdot ds}{r^2} = ds'.$$

Since ds' is an area element on the unit sphere S', we have

$$\int_S \frac{\hat{r}\cdot ds}{r^2} \int_{S'} ds' = 4\pi.$$

The surface boundary of a volume A, is written ∂A. Therefore

$$\int_{\partial A} H\cdot ds = -\int_{\partial A} \frac{GM\hat{r}\cdot ds}{r^2} = -GM4\pi.$$

For the case of a distributed mass, with density function ρ, each mass element $dm = \rho dv$ produces a contribution to the field dH, where dv is the volume element. Then

$$\int_{\partial A} dH\cdot ds = -G\rho 4\pi dv.$$

When we integrate, we get

$$\int_{\partial A} H\cdot ds = -4\pi G \int_A \rho dv.$$

This is the integral form of Gauss's law for gravitational mass. It says that the surface integral of the field is equal to $-4\pi G$ times the amount of mass inside the surface. In the next section we shall introduce the concept of the divergence, $\nabla\cdot H$, of a vector field. Then the differential form of Gauss's law is

$$\nabla\cdot H = -4G\pi\rho.$$

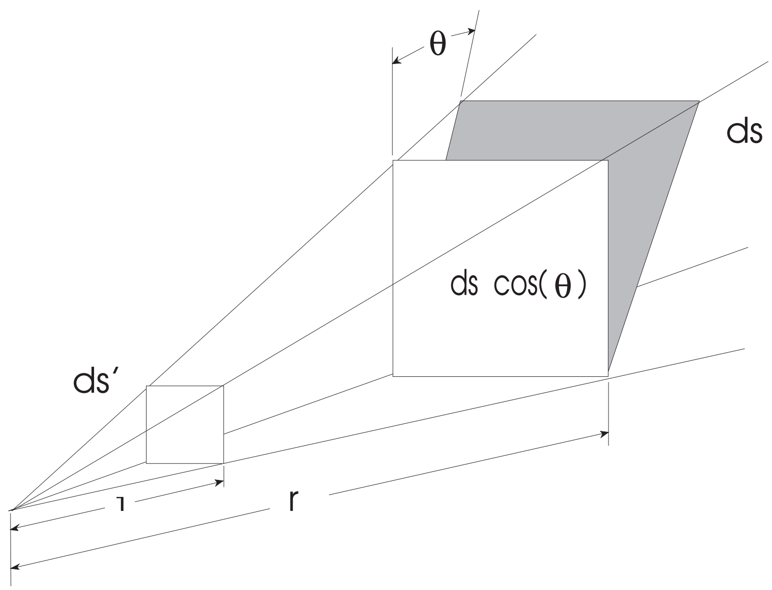

Figure 1: Proof of Gauss' Law.

6 Laplace's Equation

In a region where there are no charges we have

$$\nabla \cdot \mathbf{E} = 0.$$

We have that the electric field is equal to the negative gradient of the potential ϕ.

$$\mathbf{E} = -\nabla \phi.$$

Thus we obtain

$$\nabla \cdot \mathbf{E} = \nabla \cdot (-\nabla \phi) = 0.$$

So the potential satisfies Laplaces's equation.

$$\nabla^2 \phi = 0.$$

7 Surface Charge

Let a conductor have surface charge σ. Take a small pill box enclosing a small surface element dS. One side of the pillbox is inside of the conductor where the electric field is zero. We can take the sides of the pillbox to be of arbitrarily small area. So by Gauss's law, we have the normal component of $\mathbf{E}$ is

$$E_n = \frac{\sigma}{\epsilon_0}$$

8 Coefficients of Potential

Given n conductors we define p_{ij} to be the potential of conductor i when there is unit charge on conductor j and the other conductors are uncharged.
Proposition. If a potential is multiplied by a constant c, then the charges are multiplied by c.
Proof. Use $E_n = \sigma/\epsilon_0$, and $\nabla\phi = -\mathbf{E}$.

From this proposition, $Q_j p_{ij}$ is the potential on conductor i, when Q_j is the charge on conductor j and the other charges are zero.

In the general case, because the potential satisfies Laplace's equation, by linear superposition, the potential on conductor i is

$$\phi_i = \sum_{i=1}^{n} p_{ij} Q_j$$

when the charges are Q_j, j=1...n .

The energy of the conductors is

$$U = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} p_{ij} Q_i Q_j.$$

The coefficients of potential are symmetric,

$$p_{ij} = p_{ji}$$

This may be shown by using the expression for the energy of the conductors and taking the differential of the energy. Suppose only the charge Q_1 is nonzero. We get

$$dU = \frac{1}{2}\sum_{j=1}^{n}(p_{1j} + p_{j1})Q_j$$

This is also equal to

$$\phi_1 dQ_1 = \sum_{j=1}^{n} p_{1j} Q_j dQ_1.$$

Equating these two expressions, we find that

$$p_{1j} = p_{j1},$$

and so in general

$$p_{ij} = p_{ji}.$$

The coefficients of potential are positive (Reitz and Milford, 3rd ed., p 121).

Suppose there are only two conductors in a capacitor. The charges are equal, thus

$$C = \frac{1}{p_{11} + p_{22} - 2p_{12}}.$$

When

$$\phi_i = \sum_{i=1}^{n} p_{ij} Q_j$$

is inverted, we get

$$Q_i = \sum_{i=1}^{n} c_{ij} \phi_j.$$

The c_{ij} are called the coefficients of capacitance, and are elements of a symmetric matrix, being the inverse of a symmetric matrix. This follows by the finite spectral theorem. A symmetric matrix can be diagonalized by an orthogonal transformation, that is the eigenvectors of a symmetric matrix are orthogonal.

9 Capacitance

If C is the capacitance of a capacitor, we have

$$V = \frac{Q}{C},$$

Where V is the potential or voltage, and Q is the charge on each conductor.

In electrostatic demonstrations the charges are small. But if the capacitance is also very small, then the voltage V can be very large. The Leyden Jar is a capacitor of small capacitance, with conductors having relatively large separation, which prevents discharge with high voltages.

10 The Electrophorus

http://www.ece.rochester.edu/~jones/demos/electrophorus.html

http://www.exo.net/~pauld/summer_institute/summer_day14electrostatic/Electrophorus.html

11 The Wimshurst Machine

http://www.youtube.com/watch?v=KidNSdGqaFE

12 The Tesla Coil

The tesla coil usually consists of a high voltage, (a few thousand volts) source, say a neon sign transformer, in series with a capacitor, a spark gap, and a primary winding of the tesla transformer, which usually consists of a a low number of turns of a flat coil often made of copper tubbing. This coil surrounds a column upon which is wound many turns of magnet wire forming a high voltage coil. The bottom of this high voltage winding is connected to a grounding wire. The top of this winding is connected to a large torroidal cap. The spark gap is made of tungsten. When the power is turned on, the current jumps the spark gap, ionizes the air and causes a high frequncy discharge in the primary circuit of the tesla coil. The top cap of the very high voltage column functions as a capacitor with charges being separated between the metal cap and the ground. This high voltage discharges from the metal cap to the air and hence the ground. The high voltage generates is a LRC discharge at very high frequency.

http://www.youtube.com/watch?v=FY-AS13fl30
http://tesladownunder.com/Tesla18Dalek10003Ft.jpg

A tesla coil can be made to play music. The music signal operates a switch that interupts the high voltage in the primary as a sort of digital modulation.

13 The Cockcroft-Walton Accelerator: Voltage Multiplication Circuits

Wikipedia **Cockcroft-Walton Generator** c:/je/pdf/cockcroft-waltongenerator.pdf

14 The Van De Graff Generator

See Make Magazine, Volume 28, p.124, fall 2011, **Simple Van de Graaff Generator** by Adam Wolf. A generator made of a soda can, a large rubber band, and PVC pipe.

http://en.wikipedia.org/wiki/Van_de_Graaff_generator
http://www.google.com/patents?vid=1991236

15 Charles Proteus Steinmetz

Ronald R. kline, Steinmetz: Engineer and Socialist,
John Hopkins University Press, 1992

Rudolf Eickemeyer manufacturer of dynamos, Otis elevator moters, Westinghouse, Edison company, AC vs DC, Breslau, Zurich, General Electric, induction motors ,hysteresis, complex numbers for for alternating currents, a+bj , socialism, From athematician to Engineer. Panic of 1893, 5000 of 8000 laid off, 1898 boarding house quasi -Bohemian unusual animals monkeyaligator sister Claa poet and painter Bergs barn Societ for the Adjustments in Differences in Salary, poker. liberty hall calculating department

Gabriel Kron GE in the 1930's, p116 "Steinmetz" kline

Silvanus Thompson

Wizard of Schenectady, Liberty Hall, Westinghouse went to union college, how did he get to Pittsburge, business original schenectady, Edison to Schenectady to avoid labor troubles in new york. Pupin vs Steinmetz. Rational equations vs Emperical equations.

The Calculating Department.

General Electric Research Laboraty 1900

Equivalent circuit for induction motor 1887.

Taught at union college Dialectic constant of air 75000 volts per inch Lightning arrester

Socialism in Schenectady, 1916-1920. Corporate Socialism.

16 Michael Pupin

Invented the loading coil for long distance telephony. Transmission line short distance atrophy. From Serbia, graduated from Columbia University, later a PhD under Helmholtz in Germany. Professor of Physic

17 Nikola Tesla

Tesla, Nikola, 1856-1943

Tesla's polyphase patents. invention of the induction motor 1887. Arago's disk.

"My Inventions", Nikola Tesla, 1919 Magazine seriies, Edited and introduction by Ben Johnston.

18 Numerical Capacitance Calculation

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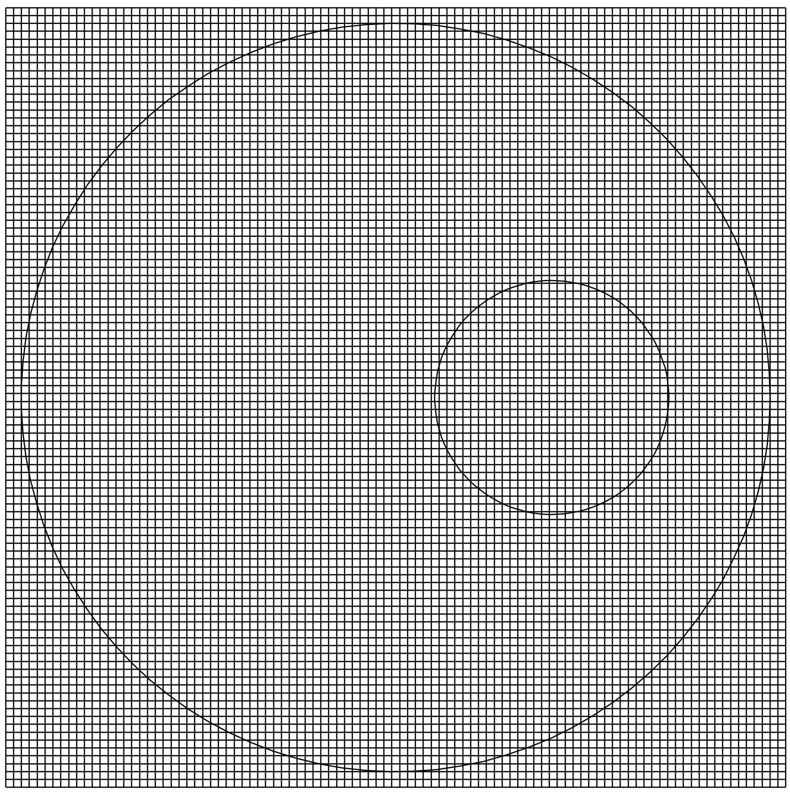

Figure 2: **Capacitance of Cylinders:**

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