STEM Society Meeting, February 14, 2017

James Emery

Last Edit: 3/1/2017

Contents

1 About the STEM Society and the STEM Society Website

STEM is an abbreviation for Science, Technology, Engineering and Mathematics. The acronym STEM is commonly associated with K-12 education, but our use of the term is only slightly bound to this meaning. There are over one hundred people on the mailing list, although a much smaller group attends any one meeting. We meet on the second Tuesday of each month at the Trailside Center at 99th and Holmes in Kansas City, Missouri. The meetings are open to all. The start time is 6PM. We make presentations, have discussions, and have demonstration experiments. These relate to Science, the History of Science, Mathematics, Engineering, Philosophy and Technology at all levels. The topics have ranged from a technical discussion of the Mathematics of General Relativity to scientific experiments for young students.

These meeting notes contain links to many other documents, which may be viewed or downloaded by clicking the link. A partial list of documents can be reached by clicking the heading **Documents**. The meeting notes may also be viewed in an archive file (archive.pdf), which is in the list of documents. Many of the documents are PDF files. They may be viewed or downloaded to the computer by clicking, provided Adobe Reader, or another program capable of reading PDF files, is present. There are many more documents available at the site than are listed under **Documents** because the documents.htm file is not at all up to date. The last time I checked, about March 2014, there were about 350 document files on the site. We are in the process of creating better techniques for finding documents and authors. The first meeting of the STEM Society was in November of 2006. For several years we used the content management program called Joomla. It had a fancy looking interface, but was hard to use. It overran the space somehow at our internet provider Bluehost. So we now have a very simple HTML site. It is not so slick looking as Joomla, but is very easy to maintain and modify.

The web site is:

`http://www.stem2.org/`

Direct to the documents list:

`http://www.stem2.org/je/documents.htm`

Direct to the archive file:

`http://www.stem2.org/je/archive.pdf`

2 The February 14, 2017 Meeting Announcement

The February meeting of the STEM Society will take place on the second Tuesday of the month, February 14, 2017, at the Trailside Center at 99th and Holmes in Kansas City, Missouri. The starting time is 6PM. Also look at our website for past meeting notes:
The web site is:

`http://www.stem2.org/`

Possible Topics and Discussions:

(a) I will talk about David Hilbert, his biography and work in Mathematics and Physics, and about the International Congress of Mathematicians meeting of 1900 where Hilbert presented his famous 23 problems, some of which we may attempt to discuss.

(b) I will bring a recently printed copy of my book "Computational Mathematics."

(c) The stem society has been going now for over 10 years, so Bob Kessler has called to my attention an interesting report on one of our first meetings, so perhaps we shall look at our past a bit.

(d) Suggested by Charlie Mentesana: A discussion: on the book by Ray Kurzweil *The Singularity is Near*, concerning computer intelligence and robotics.

(e) Suggested by Charlie Mentesana: Relativity without equations, (perhaps considering only intuition).

(f) Rich Kaufman: a presentation on a topic from Science Magazine.

(g) I hope we will get some contributions from others because it is so lonely at the top.

3 Rich Kaufman: On an Article from "Science 3 Feb 2017, pp446-450," on Moderna Therapeutics, by Kelly Servick

Rich gave some introductory remarks, introducing DNA, RNA, base coding for amino acids making up a protein, the ribosomes in the cell and the associated mechanics of protein systhesis, with an illustration.

This is an article about how biotech company **Moderna Therapeutics** delivers sythesized mRNA (messenger RNA) into cells used by the cell ribosomes to manufacture proteins which can be used to treat diseases. There are four kinds of RNA nucleosides: adenosine, cytidyne, uridine and guanosine. Sequences of these code for manufacturing protein. This would be a very effective way to manufacture drugs containing these specific proteins. However RNA invading a cell from the outside is the signal of an invading virus, so the immune system will try to destroy such mRNA invading strands. Such immune response is tamed by Moderna by modifying uridine, replacing it with psuedouridine, which occurs naturally in the body, to greatly reduces the tendancy of immune sentinals, called dendritic cells, to shoot out from inflammatory cells in response, thus protecting the mRNA from destruction. 1-methl psuedouridine contains a chemical "bump" that prevents locking into key receptors on the immune cell surface. Moderna, a Boston company was formed in 2011. In human safty trials, vaccines armed against two flue strains and the zika virus have been tested.

4 Hilbert

We discussed Hilbert's life, his work in both mathematics and physics, and his famous 23 problems. We talked specificly about the 1st problem, about the foundations of mathematics, intuitionism, sets and cardinal numbers, and about the 10th problem on constructing solutions to Diophantine Equations. See below for a little introduction to these topics.

For more see:

/stem2.org/je/hilbert.pdf

5 Jim Emery: Ideas Concerning Singularity and Intuitive Relativity

Meanings of Singularity

1. The state of being singular, distinct, peculiar, uncommon or unusual.
2. A point where all parallel lines meet.
3. A point where a measured variable reaches unmeasurable or infinite value.
4. (Mathematics) the value or range of values of a function for which a derivative does not exist.
5. (Physics) a point or region in spacetime in which gravitational forces cause matter to have an infinite density; associated with Black Holes.

I don't mean to attribute meaning (1) to Ray Kurzweil himself.

- Relativity (without equations). How do we develop a relativistic intuition? Perhaps in the same way we developed an intuition as children for classical mechanics. However, at relative speeds that we experience in our daily lives, that is at very slow speeds as compared to the velocity of light, which is about 3×10^8 meters per second, there are no relativistic effects. That is relativity gives the same results as the classical.
- We did this as children for classical mechanics in several ways:

1. By falling off the roof of the garage and landing on the prize red roses, developing bruises the color of the roses, so perhaps yelled at, but at the same time gaining experience the gravitational field.
2. By checking our watch by swinging in a tire suspended from a tree and checking the period of this tire pendulum.
3. By jumping up and down on the parents bedroom mattress learning about oscillation, and impaling ourselves on the bedpost.

4. Loosening our bicycle wheel nut by turning it counterclockwise and absorbing the concept of the right handed screw. But while playing mechanic with our bicycle we may have realized that this does not work for loosening the pedal crank, so perhaps by chance we learn a left wing strategy, an opposite method and ultimate success in loosening the crank and chain sprocket. We learn about the existence of the left handed screw.

5. By being elevated high in the air on the seesaw, we are up, and our partner on the opposite end of the plank is down. If as a prank, she decides to scoot off of her end. We learn a classical lesson in weightlessness, ... and possibly about a broken ankle.

6. By accidentally sticking a foot in the bicycle spokes while riding on the handlebars, we observe how the foot is now painfully threaded through the spokes and squeezed between the fork and the wheel, and being unable to free the foot because of a lack of courage in performing amputation with the handy girl-scout pocket knife, the friend who was peddling the bicycle, scared by the screaming, runs home and hides under the covers.

7. By playing catch with a baseball on a merry-go-round with a diametrically situated playmate while the merry-go-round is spun at a good angular velocity by the rest of the gang, while they cheerfully shout "Corioles Effect, Corioles Effect!"

(C.) In the relativistic case maybe we could learn similar relativistic intuition:

1. By switching off the satellite's general relativistic time correction (Is it the purple button?), which is needed due to the lower gravitational field at the location of the satellite, and seeing how our robot driven car controlled by GPS then crashes into a tree.

2. By certain time travel problems, such as: What to do about the social awkwardness that arises when the grandfather impregnates his girl friend then travels off at the speed of light, turns around, and returns at the speed of light, so returns younger than a grandchild; And what are the social ramifications of this behavior? Must a father younger than his child pay child support or vice versa?

3. By changing the accelerating voltage of a CRT by alternating from 20000 volts, to a trillion volts, at an appropriate frequency, to see the affect on the screen image from the relativistic variation in velocity of the electron. But getting a trillion volt accelerating voltage may be a problem, and the CRT screen may not withstand this high electron velocity assault. So alternately we might visit the Stanford Linear Accelerator under the Interstate near Palo Alto.

But relativistic intuition may be harder to come by than a classical one! Perhaps our discussion will produce more ideas on intuition about relativistic behavior. But I think solving equations might be a wiser choice for developing intuition.

6 The Continuum Hypothesis and Cardinal Numbers

The following material is from the document: Emery James, **Infinity and Set Theory**,

/stem2.org/je/infinity.pdf

6.1 Cardinal Numbers

Consider the set

$$A = \{a, b, c, d, e, f, g, h\}.$$

The number of elements in a set is called the cardinal number of the set. Any two sets A and B have the same cardinal number if there is a one to one correspondence between the elements of the two sets. This finite set A above, has cardinal number $n = 8$, because it contains 8 elements. The set of all subsets of a set B is called the power set of B and is written as $\mathcal{P}(B)$. So consider a set with three elements

$$B = \{1, 2, 3\}$$

The set of subsets of set B is

$$\{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}.$$

where $\emptyset$ is the empty set. The power set of B has 8 elements, so $\mathcal{P}(B) = 8$. So the cardinal number of A and the cardinal number of $\mathcal{P}(B)$ are equal.

For the finite set A above, the set of all subsets looks like this

$$\mathcal{P}(A) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{e\}, \{f\}, \{g\}, \{a, b\}, ..., \{a, b, c, d, e, f, g, h\}\},$$

The cardinal number of this set is $2^8 = 256$. In fact if a finite set A has cardinal number n, then the power set of A, $\mathcal{P}(A)$ has cardinal number 2^n.

To see this, let

$$A = \{a_1, a_2, ...a_n\}.$$

Let k be any nonnegative integer less than 2^n . In binary form this number may be represented with n binary digits, each digit being a 0 or a 1. For example the number $k = 5$ would be written as

$$0000...101 = 0(2^{n-1}) + ... + 0(2^3) + 1(2^2) + 0(2^1) + 1(2^0).$$

Then such a number k would map to the subset consisting of the elements of A that match up with the location of the 1's in the binary representation of A. Hence for example, if $k = 5$, then $k \to \{a_1, a_3\}$. So we see that there is a one to one, onto, mapping of the integers $0 \leq k < 2^n$ to the subsets of A. There are 2^n such integers and so 2^n subsets.

The cardinal number of the integers, $\mathbf{Z}$, is written as $\aleph_0$. Aleph is the first letter of the Hebrew alphabet. This is the smallest infinite cardinal number. Infinite cardinal numbers are called transfinite numbers. The natural numbers are the nonnegative integers. They also have cardinal number $\aleph_0$, for we can create the following one to one correspondence from the natural numbers to the integers

$$\{(0, 0), (1, -1), (2, 1), (3, -2), (4, 2), (5, -3), ...\}.$$

Similarly $\aleph_0$ is the cardinal number of the positive integers. Sets that have cardinal number $\aleph_0$ are called countable sets because the elements of the set can be labelled with the counting numbers

$$\{1, 2, 3, 4, 5, 6, ...\}$$

Notice that there is a one to one correspondence between the positive integers and the even integers. Hence the positive integers are equivalent to a proper subset of themselves. This is characteristic of a transfinite cardinal number.

The positive rational numbers are the fractions, each consisting of a positive integer numerator and a positive integer denominator. So let a rational number m/n be given. We may locate this number in a matrix in row m and column n. We can count all of these fractions by repeatedly moving up the matrix diagonals, counting only those fractions that are in lowest terms. It follows that the rationals are also countable. So one might ask, " Are there uncountable numbers?"

Cantor's Diagonal Process

Consider the real numbers between zero and 1, assuming that for now that each of these is a number represented by an infinite sequence of decimal digits. These numbers may end in an infinite sequence of zero's, but we do not allow an infinite sequence of trailing 9's. This is because for example

$$.0999999999999... = .1000000000...$$

so that there would be two representations for the number 1/10, and so on. To see that consider summing the following the following geometric series

$$\begin{aligned} &9/100 + 9/1000 + 9/10000 + ... \\ &\frac{9}{100}[1 + 1/10 + 1/100 + 1/1000 + ...] \\ &= \frac{9}{100}\sum_{i=0}^{\infty}(1/10)^i \\ &= \frac{9}{100}\frac{1}{(1-(1/10)} \\ &= \frac{9}{100}\frac{10}{9} \\ &= 1/10. \end{aligned}$$

Assume that these numbers are countable. Then the first one may be written as the first row of an infinite matrix. Then the second and so on. Now we claim there is a number that does not occur in this list. For construct a number x as follows, let the first digit be a digit that is different from the first digit of the first real number in the first row of the matrix. Let the second digit be a digit different from the second digit of the second number in the second row of the matrix. Continue in this way, making the nth digit of the number x, different from the nth digit of the number in the nth row of the matrix. It follows that the number we have constructed is different from each of the numbers represented in the rows of the matrix. Hence every number has not been counted and it follows that these real numbers are not countable. Hence the cardinal number of the real numbers is greater than $\aleph_0$. This number is usually denoted by $\mathcal{C}$, and is called the cardinal number of the continuum. This technique is called Cantor's diagonal process. So perhaps more concretely consider the decimal numbers between zero and one. If they are countable, then they can be listed from the 1, to ∞, listed as infinite decimals. Suppose the first five are as follows

```
.357193201 ...
.173956589 ...
.897027123 ...
.440936417 ...
.092763122 ...
   ...
   ...
   ...
```

Then the number formed along the diagonal of this infinite matrix with infinite rows and columns would be

```
.37796 ...
```

Suppose we change every number along this infinitely long diagonal to be a different digit, say

```
.73314 ...
```

Then the resulting infinite number is not one of the first five numbers in our infinite list. In deed after these changes of each digit along the diagonal, it cannot be any number in the list at all, because it differs by one digit from each of them. Therefore it is a number we have not counted. This contradicts our claim that every number real between 0 and 1 has been counted and listed. Therefore the real numbers are uncountable.

One can show that if A is a set then the cardinality of the power set $\mathcal{P}(A)$ is greater than the cardinality of A. See for example

[1]Dugundji James, **Topology**, Alyn and Bacon, 1966

Thus for example

$$Card(\mathcal{P}(\mathcal{C})) > Card(\mathcal{C}).$$

Thus there are many orders of infinities. Most people probably think that there is only one infinity. Thus if we call somebody infinitely great we must qualify the infinity.

Proposition

$$Card(\mathcal{P}(\mathcal{Z})) = \mathcal{C}.$$

Proof. Roughly, the set of subsets of $\mathcal{Z}$ is the set of maps of $\mathcal{Z}$ to the set $\{0, 1\}$. Each real number x in the interval $(0, 1)$ has the representation

$$x = \sum_{i=1}^{\infty} \frac{c(i)}{2^i},$$

where c is one of the maps. See Dugundji p 48 for details.

There are various set theory paradoxes. The most famous is the Russell paradox after Bertrand Russell, the philosopher and mathematician.

Russell Paradox *Let set A be the set of all sets that are not members of themselves. So for example the Set $\{1, 2, 3\}$ is not a member of itself, so is in A. Thus A is not empty. A must either be a member of itself or not a member of itself. If A is a member of itself, then it is not a member of A, which is itself. On the other hand if it is not a member of itself, then it is a member of A, which is itself. These are both contradictions.*

Something is wrong with a set theory that lets sets be defined in that way. To get around this paradox, certain axioms have been devised that forbid such sets. The Zermelo-Fraenkel axioms are one such set of axioms, and are the common ones used in mathematics. The idea around the paradox is to restrict things that can be sets.

The continuum hypothesis is that there is no cardinal number between $\aleph_0$ and $\mathcal{C}$. The generalized continuum hypothesis is that for a transfinite set A, there is no cardinal number between $Card(A)$ and $Card(\mathcal{P}(A))$. Kurt Godel proved that the generalized continuum hypothesis is consistent with the Zermelo-Fraenkel axioms of set theory. On the other hand Paul J Cohen proved that the negation of the continuum hypotheses is also consistent with the axioms of set theory. It follows that the continuum hypothesis can neither be proved or disproved using the axioms of set theory. Paul Cohen is very famous for having proved this. He received the Fields medal for this in 1966. Paul Cohen died in 2007.

Note. Richard Fridshal who was an acquaintance of mine, and who was a CAM-I representative from General Dynamics in San Diego, grew up in Manhattan, and told me that he had Paul Cohen as a classmate at the Stuyvesant High School in New York City (Stuyvesant is a famous public high school devoted to science and mathematics).

7 Diophantine Equations

This is a small section from the document called **Number Theory** by Jim Emery, which can be found as **/stem2.org/je/numbertheory.pdf** at the stem2.org website.

Diophantine (die oh fan tine) equations are equations that are to be solved by integers.

Diophantus of Alexandria is thought to have been born in the interval $[201, 215]$ AD, and is said to have lived to be about 84, so he would have died somewhere in the interval $[285, 399]$ AD.

Here is an example of a Diophantine Equation:

How many ways can 25 dollars be specified by an x number of five dollar bills and a y number of two dollar bills:

$$5x + 2y = 25.$$

Simple problems like this can be solved by brute force trying all possibilities with a computer program, in this case using Python:

```
# dollars.py
# ways of making 25 dollars with 2 and 5 dollar bills
n5=5
n2=12
w=0
for i in range(0,n5+1):
  for j in range(0,n2+1):
```

```
    s=5*i+2*j
    if(s == 25):
     w=w+1
     print " 5's =",i," 2's =",j
print " ways=",w
```

Running dollars.py

```
 5's = 1  2's = 10
 5's = 3  2's = 5
 5's = 5  2's = 0
 ways= 3
```

The gcd of 5 and 2, (5,2) is 1. So there exists integers x and y so that

$$5x + 2y = 1$$

The set of the form $5x + 2y$ is the set

$$S = \{5x + 2y : x \in N, y \in N\}$$

Choosing $x = 0$, we see that all integer multiples of 2 are in this set. Similarly all integer multiples of 5 are in this set. Let L be the smallest positive integer that is in this set. Then L is the gcd of 2 and 5, namely (2,5)=1. Thus the set of the form $5x + 2y$ is the set of all integers N.

Here is another diophantine equation that is much harder to solve:

$$x^3 + y^3 = z^3,$$

in fact impossible to solve by **Fermat's Last Theorem**, which has finally been proved after several hundred years by **Andrew Wiles.**
Theorem. The linear Diophantine equation $ax + by = n$ has a solution if and only if the greatest common divisor of a and b divides n.

8 Bibliography

[1] Nitta Hideo, **The Manga Guide to Relativity**, (Graphic Book, Johnson County Library Corinth branch, 530.11.)