STEM Society Meeting, December 12, 2017

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Last Edit: 12/22/2017

Contents

1 About the STEM Society and the STEM Society Website

STEM is an abbreviation for Science, Technology, Engineering and Mathematics. The acronym STEM is commonly associated with K-12 education, but our use of the term is only slightly bound to this meaning. There are over one hundred people on the mailing list, although a much smaller group attends any one meeting. We meet on the second Tuesday of each month at the Trailside Center at 99th and Holmes in Kansas City, Missouri. The meetings are open to all. The start time is 6PM. We make presentations, have discussions, and have demonstration experiments. These relate to Science, the History of Science, Mathematics, Engineering, Philosophy and Technology at all levels. The topics have ranged from a technical discussion of the Mathematics of General Relativity to scientific experiments for young students.

These meeting notes contain links to many other documents, which may be viewed or downloaded by clicking the link. A partial list of documents can be reached by clicking the heading **Documents**. The meeting notes may also be viewed in an archive file (archive.pdf), which is in the list of documents. Many of the documents are PDF files. They may be viewed or downloaded to the computer by clicking, provided Adobe Reader, or another program capable of reading PDF files, is present. There are many more documents available at the site than are listed under **Documents** because the documents.htm file is not at all up to date. The last time I checked, about March 2014, there were about 350 document files on the site. We are in the process of creating better techniques for finding documents and authors. The first meeting of the STEM Society was in November of 2006.

The web site is:

`http://www.stem2.org/`

Direct to the documents list:

`http://www.stem2.org/je/documents.htm`

Direct to the archive file:

`http://www.stem2.org/je/archive.pdf`

2 The December 12, 2017 Meeting Announcement

The December meeting of the STEM Society will take place on the second Tuesday of the month, December 12, 2017, at the Trailside Center at 99th and Holmes in Kansas City, Missouri. The starting time is 6PM. Also look at our website for past meeting notes:
The website is:

`http://www.stem2.org/`

Topics and Discussions:

(a) Tom Grant will give a talk titled "Dark Zone Archeology." Here is some information Tom gave me in an email about his talk back on November 3:

"I am giving a talk to the KC Archeological Society in a few months. I have prepared a presentation entitled Dark Zone Archeology. I think it would also be appropriate for the STEM Society. It is about one hour and 15 minutes long. I would be willing to give this talk at the November meeting, or if the agenda is full at a later meeting."

(b) Also I would like to give a brief presentation of my own on some familiar algorithms at the beginning of our meeting.

(c) If there is left over time we can talk about other topics. So if you have something of interest you want to talk about, by all means bring it in.

3 Jim Emery: Elementary Algorithms in Python

We examine some elementary algorithms using Python. Python is a nice language for this sort of thing because is has available calculation with integers of unlimited length. The first subject is the algorithm for long division that we learn in elementary school. We give below a document partially presented at the December meeting. This document will no doubt be altered over time and so the current document can be obtained as (which as of this writing is the same as what we present below).

`http://www.stem2.org/je/longd.pdf`

A second document on the school square root algorithm was also briefly mentioned

`http://www.stem2.org/je/schoolsquareroot.pdf`

which is titled **The School Square Root Algorithm**.

3.1 Long Division Mathematics

Here we see the mathematical algorithm of long division.

$$\frac{341235}{252} =$$

$$
\begin{aligned}
&= \frac{341}{252}(1000) + \frac{235}{252} \\
&= (1 + \frac{89}{252})(1000) + \frac{235}{252} \\
&= 1000 + \frac{89000}{252} + \frac{235}{252} \\
&= 1000 + \frac{892}{252}100 + \frac{35}{252} \\
&= 1000 + (3 + \frac{136}{252})100 + \frac{35}{252} \\
&= 1000 + 300 + \frac{13600}{252} + \frac{35}{252} \\
&= 1000 + 300 + (\frac{1363}{252})10 + \frac{5}{252} \\
&= 1000 + 300 + (5 + \frac{103}{252})10 + \frac{5}{252} \\
&= 1000 + 300 + 50 + \frac{1035}{252} \\
&= 1000 + 300 + 50 + (4 + \frac{27}{252}) \\
&= 1354 + \frac{27}{252}
\end{aligned}
$$

3.2 Long Division Procedure

The traditional computation procedure is written as

```
         1354 + 27/252
      -------
252 |341235
      -252
       ---
        892
       -756
       ----
        1363
       -1260
        ----
         1035
         -908
          ---
           27
```

3.3 More Digits

To increase the number of digits in a division simply add zeroes to the dividend and carry on the algorithm. We can locate a decimal point the same way we used to do this when using a sliderule, by writing the number in standard form using a single decimal point with a power of ten. Do this for both the divisor and the dividend. Then we can easily locate the decimal point location in the quotient.

We may carry on the calculation beyond the decimal point where we have only zeros to bring down.

So consider the remainder 27/252 = 3/28 in the previous section. We find

```
             .107142857142857142857142 and so on
         --------------------------
    252 | 27.00000000000000000000
```

Notice that the string 714285 is repeated and will be repeated forever. This is because at each division by 252 to get another digit, the remainder must be less than the divisor 252. This means that there is only a finite number of remainders so sooner or later they must repeat, also we are always bringing down a zero, so the number we divide 252 into is the remainder of the previous division with an added zero. It follows that sooner or later we must get a repeated remainder, thus the sequence of digits must repeat exactly because the calculations must be the same as the previous ones.

This proves that the infinite decimal expansion of a rational number must consist of an infinite number of repeated sequences of digits. Also we can show conversely that such an infinite sequence that eventually is a finite sequence such as 714285 repeated infinitely often, is a finite number plus an infinite sequence of such repeats. This infinite sequence of repeats is the product of the finite repeat sequence, such as 7144285, multiplied by a sum of a geometric sequence

$$\sum_{k=1}^{\infty} a^k$$

where a is a negative power of ten determined by the length n of the repeating sequence. That is

$$a = \frac{1}{10^n}.$$

This is best shown by an example.

$$.107142857142857142 85... = 1/10 + (1/100)(714285)(\sum_{k=1}^{\infty} a^k),$$

where $a = 1/10^6$. Notice that

$$714285 \sum_{k=1}^{\infty} a^k = \frac{714285}{1000000} + \frac{714285}{1000000000000} + ...$$

Recall that the sum of the geometric series where $0 < a < 1$ is

$$\sum_{k=0}^{\infty} a^k = \frac{1}{1-a},$$

so that

$$\sum_{k=1}^{\infty} a^k = \sum_{k=0}^{\infty} a^k - 1 = \frac{1}{1-a} - 1 = \frac{a}{1-a},$$

which is a rational number.

From above we have

$$.107142857142857142 85... = 1/10 + (1/100)(714285)(\sum_{k=1}^{\infty} a^k)$$

$$= 1/10 + (1/100)(714285)\frac{a}{1-a}$$

$$= 1/10 + (1/100)(714285)\frac{10^{-6}}{1-10^{-6}}$$

$$= 1/10 + (1/100)714285\frac{1}{1000000-1}$$

$$= 1/10 + (1/100)\frac{714285}{999999}.$$

The Greatest Common Divisor of the numerator and the denominator of

$$\frac{714285}{999999}$$

is 142857, so dividing out this factor we have

$$\frac{714285}{999999} = \frac{5}{7}.$$

Thus the infinite repeating number is

$$\frac{1}{10} + \frac{1}{100}\frac{5}{7} = \frac{1}{10} + \frac{5}{700}$$
$$= \frac{75}{700} = \frac{3}{28},$$

and $27/252 = 3/28$ after dividing out the common factor 25.

Notice also more generally, that this long division algorithm shows that a repeating string of digits in a decimal expansion shows it represents a rational number, and so an irrational number like π has necessarily a nonreapeating decimal expansion. And any such nonrepeating infinite decimal expansion represents an irrational number because a decimal expansion is a Cauchy sequence and must converge because the real numbers are complete.

3.4 Another Method: Finding a Rational From an Infinite Repeating Decimal

Above we have worked with the infinite repeating decimal representation for the rational number $27/252 = 3/28$. This decimal is .10714285 where the digits 714285 repeat. We write this as $.10\underline{714285}$, where the underlined digits repeat infinitely often.

Let us assume that we do not know that $.10\underline{714285} = 3/28$. We can write this as

$$a = .10\underline{714285} = 1/10 + (1/100)(.\underline{714285}) = 1/10 + (1/100)b,$$

where

$$b = .\underline{714285}.$$

So let us compute b. We have

$$1000000b = 714285 + b.$$

Thus

$$999999b = 714285,$$

and so

$$b = \frac{714285}{999999}.$$

We shall compute the GCD (Greatest Common Divisor) of the numerator and the denominator, which can be done with a computer program such as my Python program **gcd.py**, which is given in the appendix. We find that the GCD is 142857. Dividing out this factor from the numerator and denominator of 714285/999999, we find that $b = 5/7$. Thus the infinite repeating number a above is

$$a = 1/10 + (1/100)(.\underline{714285}) = 1/10 + (1/100)b =$$

$$= \frac{1}{10} + \frac{1}{100}\frac{5}{7} = \frac{1}{10} + \frac{5}{700}$$

$$= \frac{75}{700} = \frac{3}{28}.$$

3.5 A Python Computer Program to do Long Division

The program is called **longd.py**. First we describe how we run it. To run a Python program we type the word "python" followed by the program name. and then followed by any required parameters. To get some information on required parmeters we call the program without parmeters thusly:

```
python longd.py
```

We get this information:

```
Python program longd.py divides two integers.
The integers may be of arbitrary size.
dv is the divisor, dd is the dividend
 Example: dv=252, dd=341235
Usage: python longd.py dv dd
```

So now let us run the program using the numbers listed as an example:

```
python longd.py 252 341235
```

Here is the output of the program:

```
dv=  252
dd=  341235
calling decdig(dd)

number of digits in dividend=  6
index of last element in ddl lastddl=  5

returned from decdig
digit list=  [3, 4, 1, 2, 3, 5]
temporary dividend tdd=  3 nqd= 0

tdd=  34 nqd= 1

tdd=  341 nqd= 2
temporary dividend tdd= 341


entering while
dividend index nqd 2
temporary dividend tdd= 341
quotient digit qd= 1
subtraction = 252
remainder rm= 89
quotient digits ql= [1]
print the quotient  q= 1

new dividend index nqd =  3
new temporary dividend tdd=  892
leaving while

entering while
dividend index nqd 3
temporary dividend tdd= 892
quotient digit qd= 3
subtraction = 756
remainder rm= 136
quotient digits ql= [1, 3]
print the quotient  q= 13

new dividend index nqd =  4
new temporary dividend tdd=  1363
leaving while

entering while
dividend index nqd 4
temporary dividend tdd= 1363
quotient digit qd= 5
subtraction = 1260
remainder rm= 103
quotient digits ql= [1, 3, 5]
print the quotient  q= 135

new dividend index nqd =  5
```

```
new temporary dividend tdd=  1035
leaving while

entering while
dividend index nqd 5
temporary dividend tdd= 1035
quotient digit qd= 4
subtraction = 1008
remainder rm= 27
quotient digits ql= [1, 3, 5, 4]
print the quotient  q= 1354

leaving while

divisor=  252
dividend=  341235
quotient=  1354
remainder=  27
```

The program longd.py listed below has many print statements, which are used to give the steps of the algorithm. We would want to have a version also that hides these details.

Here is the listing of the program:

Note. This is a listing of the program as it appeared on the date given in the listing. As I make changes to the program I will not necessarily update the following listing because that would be quite tedious. However I will do this when the program is finalized. The current listing can be found in my directory

```
c:\je\py.
```

```
#longd.py long division of two integers
# origination Aug 31, 2017
# latest version September 1, 2017
import sys
def decdig(m):
 #divide repeatedly by 10
 #getting quotient and remainder
 d=[]
 n=0
 q=m
 while q > 0 :
  q=m/10
  r=m-10*q
  #print " q=",q, "r=",r
  m=q
  d.append(r)
  n=n+1
```

```
  #print "n=", n
#  if( n > 30 ) :
# sys.exit(0)
 #print "d=",d
 list.reverse(d)
 #print " number of digits= ",n
 #print "d=",d
 return d

#main program
print sys.argv
#print sys.argv[0]
na=len(sys.argv)
print 'number of arguments = ', na
if( na < 3):
 print "longd.py of two integers."
 print "dv is the divisor, dd is the dividend"
 print " Example: dv=252, dd=341235"
 print "Usage: python.py dv dd"
 sys.exit(0)
dv=int(sys.argv[1])
print " dv= ",dv
dd=int(sys.argv[2])
print " dd= ",dd
print " calling decdig(dd)"
print
#ddl is the list of digits of the dividend dd
ddl=decdig(dd)
nddl=len(ddl)
print " number of digits in dividend= ",nddl
lastddl=nddl-1
print " index of last element in ddl lastddl= ", lastddl
#ddl is a list of the digits of dd
print
print " returned from decdig"
#ddl is the list of digits of the dividend dd
print " digit list= ",ddl
#tdd is the temporary dividend, which is divided by dv to get the next
# digit of the quotient q.
#pq is the partial quotient determined so far.
q=0
nqd=0
tdd=ddl[nqd]
print " temporary dividend tdd= ",tdd, "nqd=",nqd
while tdd < dv :
 nqd=nqd+1
 print
 tdd=10*tdd+ddl[nqd]
 print " tdd= ",tdd, "nqd=",nqd
print " temporary dividend tdd=", tdd
# create the empty quotient digits
ql=[]
#
print

while nqd <= lastddl :
 print
```

```
 print " entering while "
 print " dividend index nqd", nqd
 print " temporary dividend tdd=", tdd
 # get next digit of quotient by dividing the temporary dividend tdd
 # by the divisor dd.
 qd=tdd/dv
 print " quotient digit qd=",qd
 # compute the remainder
 print " subtraction =", qd*dv
 rm=tdd-qd*dv
 print " remainder rm=",rm
 ql.append(qd)
 print " quotient digits ql=", ql
 q=10*q+qd
 print " print the quotient  q=",q
 nqd=nqd+1
 print
 if nqd <= lastddl :
  print " new dividend index nqd = ",nqd
  tdd=10*rm+ddl[nqd]
  print " new temporary dividend tdd= ",tdd
 print " leaving while"
print
print " divisor= ",dv
print " dividend= ",dd
print " quotient= ", q
if rm > 0 : print  " remainder= " ,rm
```

3.6 Appendix A: Summing the Geometric Series

The series

$$\sum_{k=0}^{\infty} x^k,$$

is called the geometric series.

Theorem (Geometric Series). If $|x| < 1$, then the geometric series

$$\sum_{k=0}^{\infty} x^k,$$

conveges to

$$\frac{1}{1-x}.$$

Proof. If

$$S = \sum_{k=0}^{n} x^k$$

then

$$Sx = S - 1 + x^{n+1}$$

$$S(1-x) = 1 - x^{n+1}$$

or

$$S = \frac{1 - x^{n+1}}{1-x}$$

If

$$|x| < 1$$

then

$$\frac{x^{n+1}}{x-1} \to 0$$

as $n \to \infty$. So

$$\sum_{k=0}^{\infty} x^k = \frac{1}{x-1}.$$

This completes the proof.

You might ask, "Why is this series called the geometric series?" I came across the following explanation:

If a and b are two numbers then $\sqrt{ab}$ is called the geometric mean, a kind of average of a and b. This corresponds to a classic problem in the time of the ancient Greeks called **squaring the rectangle**. So given a rectangle of sides a and b the problem is to find the side of a square that gives the same area as the rectangle. The Greeks solved this problem with a geometric construction. They showed how to construct a square with side $\sqrt{ab}$. This is the geometric mean of the sides of the rectangle.

Now if the kth term of a geometric series is a^k, then the preceeding term is a^{k-1} and the following term is a^{k+1}. So the geometric mean of the preceeding term with the following term of a geometric series is

$$\sqrt{a^{k-1}a^{k+1}} = \sqrt{a^{2k}} = a^k,$$

so a term is the geometric mean of its two neighboring terms. Perhaps this is the origin of the ancient name "geometric series."

3.7 Appendix B: Calculation Examples

Example 1

```
c:\>python longd.py  2468024680358765234567899 12345678901234567890101001000100001000001000000100000001
```

```
divisor=    2468024680358765234567899
dividend=   12345678901234567890101001000100001000001000000100000001
quotient=   5002250990229130990790572966l225
remainder=  141431559287332036934476
divisor*quotient+remainder =  12345678901234567890101001000100001000001000000100000001
```

Example 2

Given the repeating decimal

$$b = .123456789123456789\underline{123456789}...,$$

what rational number does it represent?

We have

$$10^9 b = 123456789 + b,$$

so

$$b(10^9 - 1) = 123456789$$

$$b = \frac{123456789}{10^9 - 1} = \frac{123456789}{999999999}$$

Now we calculate the gcd

```
c:\>python gcd.py 999999999 123456789
 m=  999999999
 n=  123456789
  999999999 = 8 * 123456789 + 12345687
  123456789 = 9 * 12345687 + 12345606
  12345687 = 1 * 12345606 + 81
  12345606 = 152414 * 81 + 72
  81 = 1 * 72 + 9
  72 = 8 * 9 + 0
 v = [gcd,alpha,beta]= [9, 1524159, -12345688]
 gcd(m,n)=  9
 alpha*m+beta*n=  9
 m/gcd= 111111111
 n/gcd= 13717421
```

So this infinite decimal equals the rational number

$$\frac{13717421}{111111111}.$$

To check this we use the long division algorithm

```
c:\>python longd.py 111111111 13717421000000000000000000000000000000000000

 dv=  111111111
 dd=  13717421000000000000000000000000000000000000
 calling decdig(dd)

 number of digits in dividend=  44


 divisor=  111111111
 dividend=  13717421000000000000000000000000000000000000
 quotient=  123456789123456789123456789123456789
 remainder=  13717421
 divisor*quotient+remainder =  13717421000000000000000000000000000000000000
```

4 Tom Grant: Dark Zone Archeology

Dark Zone Archeology concerns Archeology work done underground in caves. This is a relatively new research area, and was originally scoffed at somewhat by more traditional Archeologists, but has made some tremendous discoveries. Tom Grant has been involved in Spelunking since he was quite young and has worked with many distinguished researchers over the years including the vary prominant Patty Jo Watson. Tom Grant and Rick Hines, both members of the STEM Society, are both very well known members of the United States spelunking community. Rick was also at the meeting. Tom mentioned outstanding work done by Rick in connection with the Carroll Cave, which is a Missouri cave. It turns out that caves naturally preserve ancient human remains and other artifacts, and several famous human fossils have been discovered among these are ... Tom mentioned the dangers of underwater spelunking and mentioned several of his friends that have died pursuing this dangerous activity. Very sad stories.

The following section is from Wikipedia captured on December 13, 2017.

4.1 Patty Jo Watson

Patty Jo Watson (born 1932)[1] is an American archaeologist renowned for her work on Pre-Columbian Native Americans, especially in the Mammoth Cave region of Kentucky.[2] She is now Distinguished University Professor Emerita, Archaeology at Washington University in St. Louis.[3] Until her retirement in 2004, she was the Edward Mallinckrodt Distinguished University Professor of Archaeology at Washington University in St. Louis.[4]

Education

Watson earned her Ph.D. from the University of Chicago in 1959.[3] While attending the University of Chicago, she studied under Robert Braidwood.[3][4] Career

Watson devoted much of her early career to the archaeological study of the Ancient Near East.[2][3] Her husband Richard A. Watson convinced her to change her focus from Near Eastern archaeology to work in North America.[4]

Watson is a proponent of processual archaeology and has contributed greatly to that approach.[2][5]

In addition, Watson has been instrumental in applying ethnography to the archaeological record.[6] In the 1960s in Mammoth Cave, she introduced the practice of performing recreations of ancient lifeways as a method of filling in gaps from incomplete archaeological data. "She has contributed centrally to techniques for recovering carbonized plant remains from archaeological deposits and to understanding the independent origin of pre-maize agriculture in pre-Columbian eastern North America. "[6] Her work on the diet of Native Americans who lived in Mammoth Cave has included examining the intestines of bodies found in the cave and has been notably interdisciplinary in scope.[4] Accolades

In 1988, Watson was elected to the National Academy of Sciences.[4] In its November 2002 issue, Discover included Watson among "The 50 Most Important Women in Science."[7] The article credited Watson with "establishing the best qualitative and quantitative data for an early agricultural complex in North America" and with helping to "introduce the scientific method into archaeological studies."[7] Watson received the Gold Medal Award for Distinguished Archaeological Achievement in 1999 from the Archaeological

Institute of America.[8]

References

"Watson, Patty Jo (1932-) - People and organisations". National Library of Australia. Retrieved 31 May 2017. Lindsey Alston, ed. (2007).

"Patty Jo Watson". EMuseum. Minnesota State University, Mankato. Retrieved 2009-08-19.

"Patty Jo Watson Faculty Home Page". Washington University in St. Louis. Archived from the original on October 24, 2008. Retrieved 2009-08-19.

"About Alumni: C. Vitae: Cave Crawler". University of Chicago Magazine. 95 (5). June 2003. ISSN 0041-9508. Retrieved 2009-08-26.

Herst, K. Kris. "Patty Jo Watson". About.com. Retrieved 2009-08-19.

"Academy Fellows: Patty Jo Watson, Ph.D". Academy of Science-St. Louis. Retrieved 2009-08-19.

Svitil, Kathy A. (November 1, 2002). "The 50 Most Important Women in Science". Discover. ISSN 0274-7529. Retrieved 2009-08-26.

http://www.archaeological.org/webinfo.php?page=10100. Retrieved 2010-2-12.

4.2 Mammoth Cave

Mammoth Cave is in South Central Kentucky about 90 miles Northeast of Nashville Tennessee.

4.3 Salt Cave

Salt Cave is in Kentucky about 45 miles Southwest of Mammoth Cave.

4.4 Carroll Cave

Carroll Cave is in Missouri about 45 miles West of Rolla, Missouri, near the town of Richland Missouri.

5 Tom Grant Power Point Slides

We don't have these slides now, but Tom is supposed to bring them to us on a flash drive at the next STEM Society Meeting in January.