The Penrose Tribar.

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1 Introduction

The Tribar is associated with the English mathematician and physicist Roger Penrose, who popularized it in the 50's and 60's. It was first created by the Swedish artist Oscar Reutersvrd in 1934. Penrose inspired Esher to create his famous endless staircase. Roger Penrose has worked with Steven Hawking and has been honored for his work on singularities in General Relativity.

The Tribar is an example of an impossible figure. Impossible figures were made famous by the artist Maurits Cornelis Escher. The tribar also demonstrates 3-fold rotational symmetry: Each time the figure is rotated 120 degrees it is mapped to itself. This rotation group has three elements, a rotation by 120 degrees, a rotation by 240 degrees, and the identity, which is a rotation by 360 degrees.

2 Drawing The Tribar Figure by Computer

The figures were drawn with program **tribar.ftn**. This is done by creating commands for draws and moves for my graphics language called "eg." An "eg" file is a text file. The "eg" file is converted to Postscript with my program **eg2ps.exe**, whose source is a **C** program. For more information about this see my document called **figures.pdf**.
Note to Myself See my notebook, 6/10/2011, page 23.

3 Drawing A Tribar Figure by Hand

`http://www.stem2.org/je/drawtribar.pdf`

(1) Draw an equilateral triangle.

(2) Draw a narrow 60 degree parallelogram along the bottom edge of the equilateral triangle. Rotate the paper by 120 degrees and again draw a narrow 60 degree parallelogram along the bottom edge of the equilateral triangle. Repeat one more time.

(3) Fill in the vacant corners with equilateral parallelograms.

(4) Draw lines inside the equilateral triangle parallel to the sides in such a manner that each corner of the tribar looks like three dimensional joined blocks.

Note. The equilateral triangle and the narrow parallelograms can be constructed accurately, by drawing circles and using the fact that laying off radii along the circumference gives the vertices of an equilateral triangle.

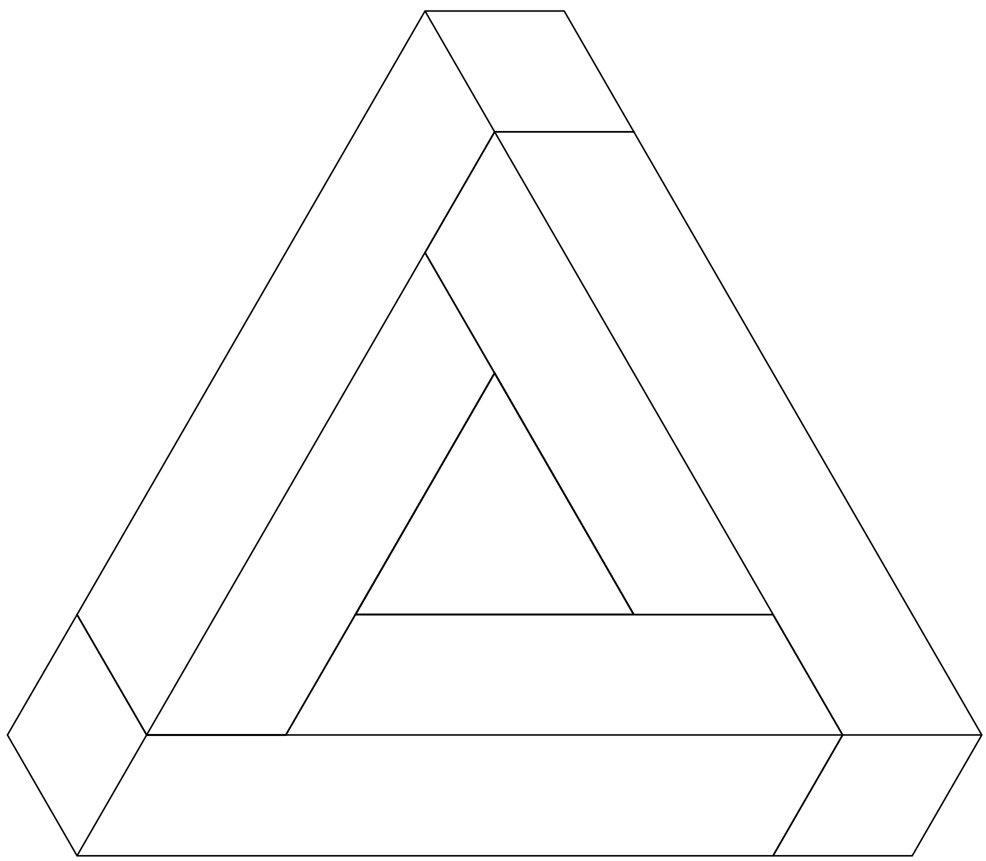

Figure 1: The Tribar is an example of an impossible figure. Such figures were made famous by the artist Escher. The tribar also demonstrates 3-fold rotational symmetry: Each time the figure is rotated 120 degrees it is mapped to itself. This rotation group has three elements, a rotation by 120 degrees, a rotation by 240 degrees, and the identity, a rotation by 360 degrees.

4 A Stroboscopic Effect

If one third of the tribar figure is mounted on a rotating disk, and illuminated by a stroboscope operating at three times the frequency of rotation of the disc, then a completed image will be created by our eyes and brain. Persistence of vision will allow us to see all three images merged together as one image, thus demonstrating the rotational symmetry of the tribar figure..

5 The Tribar Puzzle

The tribar may be constructed from just three polygon shapes, two different parallelograms, and a trapezoid. So the tribar may be assembled from nine polygons, which can be cut out of masonite board, and colored. The puzzle is to construct the tribar from these nine polygons scattered on a table top.

6 What is a Group?

Groups that pertain to symmetry are transformation groups. So elements of the group are transformations of a space of points. A product of group elements is a composition of transformations. So if T_1 takes point p to point q, we write this as

$$T_1(p) = q.$$

Suppose T_2 takes q to r.

$$T_2(q) = r.$$

The group product of T_2 and T_1, written as T_2T_1, would be defined as

$$T_2T_1(p) = T_2(T_1(p)) = T_2(q) = r.$$

7 Formal Definition of a Group

Groups first appeared in mathematics as sets of roots of polynomial equations. This was introduced by the famous French mathematician Galois, who created Galois Theory but did not realize it at the time. He died in a dual at 21 and never actually heard of Galois Theory.

Not all groups are necessarily transformation groups. The theory of groups originated as the study of the roots of polynomial equations. The

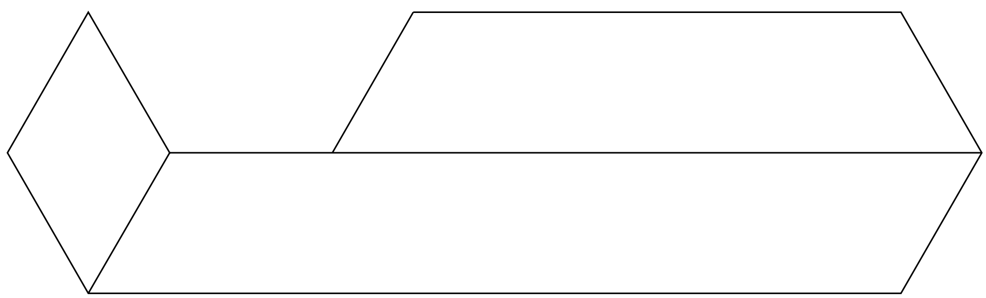

Figure 2: The Tribar can be generated by rotating this figure twice.

"group" was the set of roots, in particular it was a set of permutations of these roots. That is, it was what we would now call a permutation group. For a nice outline of the historical classical theories of the roots of equations, see, **Mathematics: Its Content, Methods, and Meaning,** Volume I, pp 255-280, by Aleksandrov, Kolmogorov, Laurent'ev, translated by S H Gould et al. This includes Cardan's formula for the cubic, Lagrange's methods, the Lagrange resolvent, Abel's proof of the impossibility of solving the general equation of the 5th degree and higher by radicals, and a good outline of Galois theory.

Definition An abstract group G is a set with a binary operation defined between pairs of elements of the set. The operation is associative. For a, b, and c in G, $(ab)c = a(bc)$. A group G has an identity element e so that for every element g of G, $eg = ge = g$.

Every element g of G has an inverse g^{-1}, so that $gg^{-1} = e$ and $g^{-1}g = e$.

8 Group Theory and Symmetry

Group Theory and Symmetry play a very important roll in parts of Physics and Chemistry. This includes Quantum Mechanics, Particle physics, and the structure of molecules. Specifically in Particle Physics the concept of Lie Groups (pronounced "Lee") is central to the subject. Lie Groups were originally called Continuous Groups of Transformation. They are algebraic and at the same time topological.