Baby Geometry, By Zeus!

James D Emery

3/27/2012 Edition

Contents

0.1 The Carpenter's Lemma

As we all discovered in our careers as carpenters, that the steel square is a computing device of great power. If we pound two nails into a board, and rotate our square around against the nails, the vertex of the square describes a circle, with the line joining the two nails a diameter.

0.2 Pythagoras Plays With Four Triangles

One day Pythagoras made a triangle. He was very conservative, so he called it a right triangle. He called the long side a, the short side b, and the side opposite the right angle c. He was so pleased with himself, that he made three more. He began to play with his triangles. He found that he could arrange his four triangles into a square with side c. But in the center was an empty square hole. So he made a small square to fill in the hole. So he had a square of area

$$A_1 = c^2.$$

Then he thought, "I wonder what the area is of the pieces." He knew what the area of a rectangle was, namely the length times the width, which is pretty much the definition of area. But he also had four triangles. What is the area of these? He knew that the area of a figure can be computed as an integral of a function $f(x)$,

$$A = \int f(x)dx,$$

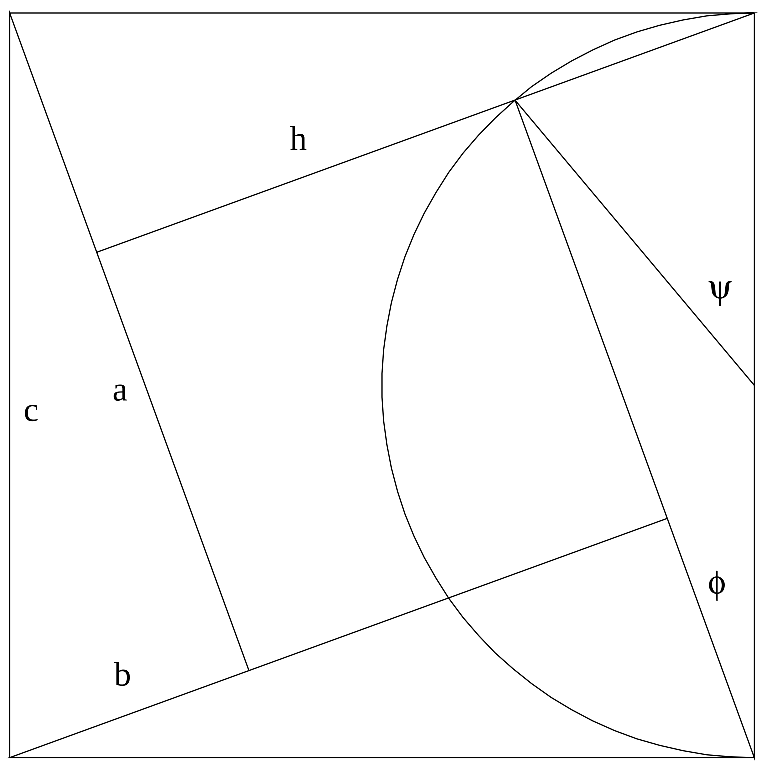

Figure 1: The Pythagorean triangles of sides a, b, c, fitted into a square of side c. The triangle is specified with an angle ϕ. The triangle is drawn using the angle $\psi = 2\phi$. The inner square has side $h = a - b$

but they had not yet invented functions, so he tried another method. He reasoned that if one put two identical triangles together, he would get a rectangle. So the area of the triangle must be one half of the rectangle! So the area of each of his four triangles must be

$$\frac{1}{2}ab.$$

Now the side of the inner square must be $a-b$, and so its area is $(a-b)^2$. His friend Mohammed, who was shortly to go off and invent a religion (perhaps my chronology is a little shaky), helped him with some algebra. So now he computed the total area of his pieces, which form a square of area c^2, as

$$A_2 = 4(\frac{1}{2}ab) + (a-b)^2$$

$$= 2ab + a^2 - 2ab + b^2$$

$$= a^2 + b^2.$$

But the two areas A_1 and A_2 must be the same. He hit himself violently in the forehead with his palm, and exclaimed, "By God (Being Greek, he actually said "By Zeus"), I have proved the Pythagorean Theorem!" and he wrote

$$a^2 + b^2 = c^2.$$